Economic and policy uncertainty: Aggregate export dynamics and the value of agreements☆

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We examine the interaction of economic and policy uncertainty in a dynamic, heterogeneous firms model. Uncertainty about foreign income, trade protection, and their interaction dampens export investment. Trade agreements can mitigate uncertainty and the probability of policy uncertainty shocks. We use firm data from 2003–2011 to establish new facts about U.S. export dynamics. These facts include a differentially lower export growth to non-preferential markets driven by the extensive margin at the start of the Great Trade Collapse. The structural model can explain this differential as a consequence of a shock to demand uncertainty. The shock was three times larger for non-preferential markets than preferential ones and it was later reversed, which is consistent with the timing of trade war fears in the crisis.

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1. Introduction

Uncertainty increases during downturns and a growing literature examines the effects of either economic or policy uncertainty shocks (Bloom, 2014). The interaction between these shocks may amplify uncertainty: for example, government actions to ameliorate downturns can increase policy uncertainty (Pastor and Veronesi, 2013). This is one reason policymakers attempt to commit to predictable policy regimes. We provide a model for the interaction of economic and policy uncertainty and a motive for governments to address it using trade agreements. We focus on trade policy given its international externalities, responsiveness to economic and political shocks (cf. Bown and Crowley, 2013b), and because trade can expose industries and firms to more foreign volatility (di Giovanni and Levchenko, 2012; Fillat and Garetto, 2015). We estimate the model using data on firms’ international trade decisions during the 2008 recession and recovery—a period when international trade collapsed and fear of a trade war was initially widespread.

The impact of uncertainty on firm investments is theoretically understood (cf. Bernanke, 1983; Dixit, 1989) and there is growing empirical evidence for this mechanism (Bloom, 2009, 2014). Businesses, policymakers, and economists often discuss instances of policy uncertainty, but evidence for the role it plays in firm decisions remains limited. The lack of evidence stems in part from the difficulties with measuring policy uncertainty, identifying its causal impact on specific investment decisions (Rodrik, 1991), and disentangling it from economic uncertainty.

The international trade context is well suited to evaluate and quantify the importance of economic and policy uncertainty. Using firm-level trade transactions data to identify investments in market entry and exit, we can better match theoretical predictions to data and estimate and quantify the effects of economic and policy uncertainty. sunk costs of exporting affect market entry (cf. Roberts and Tybout, 1997) and can generate a higher option value of waiting to enter when uncertainty increases. Adverse foreign income shocks are especially salient for exporting firms; they may trigger changes to trade policy and protectionist remedies in the foreign market. There is evidence that agreements increase aggregate bilateral trade (cf. Baier and Bergstrand, 2007; Subramanian and Wei, 2007) and that one important channel is reductions in trade policy uncertainty (TPU) working through increased export investments (Handley, 2014; Handley and Limão, 2015; Feng et al., 2017). There is scant evidence of the insurance role of these agreements during periods of economic volatility when trade wars are more likely.

We address the following questions: How do international economic and trade policy uncertainty affect firms’ trading decisions? What was the role of this uncertainty during the Great Trade Collapse (GTC) and subsequent recovery and how was it affected by international trade agreements? We address these as follows. First, we document the dynamics of U.S. exports during the GTC using aggregate and firm-level data. Second, we develop a model consistent with the main features of the data. Third, we develop an approach to identify export shocks using aggregate data that we use to quantify the impacts of foreign uncertainty shocks on aggregate U.S. exports and how they are mitigated in preferential markets.

We explore U.S. export dynamics during the GTC and note three key patterns in the data that we relate to an uncertainty mechanism in the subsequent model. First, while the export collapse from 2008Q3 and 2009Q2 was dramatic, it was followed by a quick, partial recovery. Measures of export values and trade participation return to their pre-crisis peak by the end of 2011. Second, both the intensive and extensive margins played important roles. The extensive margin—the creation and destruction of varieties (measured by bilateral firm-country-products)—contributed to about one third of the contraction in U.S. export growth, a larger role than has been documented for other countries. The number of exporting firms declined about 9% and the number of varieties declined 11% between 2008Q3 and 2009Q2. Third, there was notable heterogeneity across countries. Firms adjusted less through the extensive margin towards preferential trade agreement (PTA) countries relative to non-PTA countries. Standard economic determinants can predict most of the decline in aggregate exports via the extensive margin to PTAs in the first year of the crisis, but fail to do so for non-PTAs, where an additional 4.6 pp reduction is unexplained. There is no such differential across markets for the intensive margin.

Our focus on modeling the interactions of policy and economic uncertainty is motivated in part by international efforts to promote policy cooperation following the economic shocks of the financial crisis and GTC. Several factors suggest the crisis increased uncertainty about future protection. First, there was widespread discussion of a trade war—similar to those in the 1930’s triggered in part by the Depression—that prompted assurances by members of the G-20 and other institutions that “We will not repeat the historic mistakes of protectionism of previous eras.” Second, increases in import protection often follow economic downturns (Bown and Crowley, 2013b). Early in the crisis, protectionism seemed likely given government interventions to stimulate markets while discriminating against foreign firms, e.g. the “Buy American” clause in the U.S. stimulus bill (Eichengreen and Irwin, 2010).

The threat of a trade war was present during the GTC. But because the threat was never realized, it makes this a particularly interesting setting to study TPU and heterogeneity across agreements. There were limited increases in protection during the GTC; the barriers applied affected only 1% of traded products and accounted for less than 2% of the observed collapse (Kee et al., 2013). This is in sharp contrast with the Great Depression, where increases in barriers affected 35% of tariff lines and accounted for a large fraction of the trade contraction (Madsen, 2001). One difference is that, in 2008, countries were more constrained through agreements. But the constraints and incentives to raise protection were heterogeneous across countries: WTO members had room to legally increase import protection whereas PTA partners were more constrained or less willing to do so.

These observed trade patterns and the policy backdrop during the GTC inform our model of multiple and interacting sources of uncertainty. We extend Handley and Limão (2015) in three ways that are central for the analysis of the GTC. First, we introduce demand uncertainty arising from trade policy and economic conditions (e.g. aggregate income). Second, we derive the policy uncertainty.
preferences of a government in terms of overall foreign demand uncertainty and then examine how it can be achieved using agreements that can only affect policy uncertainty. Third, we examine the dynamics of exporting more broadly, including export exit and re-entry, and derive exact relationships that map observable data, on aggregate exports and variety counts, to a set of sufficient shocks that can be used for quantitative analysis.

The basic mechanism relies on the interaction of firms’ sunk costs to start exporting and foreign demand uncertainty. Shocks to export market access, i.e. to income or trade policies, arrive with a probability $\gamma$ after which a new level of demand is drawn from a distribution of market access. Exporting to a country with riskier market access increases the option value of waiting to enter and lowers the number of exporters and aggregate exports. A government that is export risk averse will gain from a PTA that reduces overall export risk, i.e. where market access is drawn from a distribution that second-order stochastically dominates (SSD) the original market access distribution. However, agreements can only restrict the distribution of a subset of shocks affecting export conditions, e.g. trade policies. We define these as restricted agreements; characterize those which achieve an overall reduction in export risk; and argue they are consistent with real-world agreements. Moreover, we show these agreements can be more effective in reducing export risk if there are other economic shocks, e.g. to income, that are positively correlated with trade openness.

We can examine the model’s usefulness in understanding export dynamics in different ways. Carballo et al. (2018) employ a regression approach to determine if US industry export growth margins evolved differently towards non-PTA markets and how this depended on income risk. In this paper, we employ a structural approach to evaluate the performance of the model in decomposing aggregate export margins and quantify the importance of the uncertainty mechanism. To do so, we first derive dynamic export and entry equations and show that observable export values and varieties can be perfectly explained by shocks to two aggregate statistics. We use these expressions to extract the shocks implied by the data in the crisis.

We do the following with these extracted shocks. First, we impose them on the model and find that they can closely account for the untargeted dynamics of the extensive and intensive margins documented in Section 2. Second, the shocks imply that uncertainty changed during the crisis. The changes in uncertainty included a large increase in the export risk parameter in 2008Q4–2009Q3 for non-PTA destinations that was three times larger than the one towards PTAs and reversed starting in 2009Q4. We argue that this is consistent with a change in policy parameters and the timing of trade war threats that was high at the start of the crisis and later subsided. Finally, we provide counterfactual export paths without uncertainty shocks. This yields the model’s uncertainty contribution to the extensive margin, which tracks the path of the unexplained residuals in Section 2. Quantitatively, the uncertainty shocks contributed $-3.4$ pp to non-PTA growth in the first crisis year, whereas the unexplained extensive margin residual was $-4.6$ pp.

Our findings suggest that PTAs provide insurance against the threat of trade wars relative to WTO membership alone. More broadly, the paper contributes to our understanding of (i) policy flexibility and potential protectionism over the business cycle (Bagwell and Staiger, 2003; Barattieri et al., 2021); (ii) the role of trade policy commitments in the presence of economic and lobbying shocks (Amador and Bagwell, 2013; Beshkar et al., 2015; Limão and Maggi, 2015); and (iii) the implications of potentially high tariffs if cooperation breaks down (Ossa, 2014; Nictia et al., 2018).

We structure the paper as follows. In Section 2 we provide descriptive evidence of U.S. exporter dynamics in 2003–2011. These facts inform the theory in Section 3 and we combine both in the quantification provided in Section 4.

2. U.S. exporter dynamics and the great trade collapse

We characterize U.S. firms’ export dynamics and decompose aggregate exports into intensive and extensive margins of adjustment. We document a differential export growth to PTA destinations driven by the extensive margin and discuss its plausibility in the context of the institutional features of these agreements.

2.1. Aggregate exports

The GTC was a worldwide phenomenon with several competing explanations for its magnitude including: (i) changes in the composition of demand (Eaton et al., 2016); (ii) the collapse of trade credit (Amiti and Weinstein, 2011; Chor and Manova, 2012); (iii) the disintegration of international supply chains (Bems et al., 2011); (iv) the inventory cycles of firms (Alessandria et al., 2010); and (v) economic uncertainty (Novy and Taylor, 2020; Greenland et al., 2022 forth.). Our contribution to this literature is to establish the role of the extensive margin for U.S. exports and whether uncertainty played a role in the collapse and subsequent partial recovery.

U.S. exports contracted by 22% between 2008Q3 and 2009Q2 whereas its GDP contracted by 3%. Most research has focused on the collapse, but U.S. aggregate exports started expanding again by the end of 2009. However, the initial decline was so large that it took until 2010Q4 for exports to recover to their pre-crisis peak.

The collapse was especially large for some countries. Fig. 1 provides an initial piece of evidence that PTAs provided some insurance for U.S. exporters. We fit a local polynomial mean through U.S. cumulative bilateral export growth to PTA and non-PTA destinations. The average growth relative to 2002 behaves similarly for both groups until the financial crisis in 2008Q4 (solid red line). Afterwards, PTA exports decline by slightly less, recover to the pre-crisis peak earlier, and ultimately have higher cumulative growth.

We can test if the differential can be explained by a gravity regression for U.S. exports. We control for destination fixed effects by calendar-quarter, $\alpha_t$, GDP and its deflator. We also control for any common shocks to U.S. exports using time effects, $\alpha_t$. We
then estimate differential exports to PTA destinations over different periods, $\gamma_p$, where each period contains four quarters starting from 2002Q4–2003Q4 to 2010Q4–2011Q3.\(^2\) The baseline period, $p=0$, is the one preceding the collapse: 2007Q4–2008Q3.

$$\ln X_{it} = \sum_{p=-5}^{3} \gamma_p[I(p \neq 0) \times \text{PTA}_i] + \beta_y \ln GDP_{it} + \beta_d \ln GDPdef_{it} + \alpha_t + \alpha_iQ + \epsilon_{it}$$  \hspace{1cm} (1)

In Fig. 2, we plot the estimated coefficients. The PTA differential is similar to the baseline period before the crisis, but it rises and becomes significantly different from zero after the crisis starts.

\(^2\) We define the PTA indicator based on a country's status in the baseline period listed in the appendix and keep it constant in the regression. Therefore, the crisis differentials do not reflect changes in PTA status. In the pre-baseline period, there are new PTAs, e.g. Australia, which the regression treats as PTAs since 2002Q4. This may account for the lower (but insignificant) pre-crisis differential.
2.2. Firm entry and exit dynamics

Aggregate U.S. exports reached their pre-crisis peak in the second quarter of 2008 after several years of sustained growth. In Fig. 3 we plot several different measures of the extensive margin of exports. We define varieties as a firm-country-HS10 product triplet. By 2009Q1 the number of varieties exported by the U.S. to all destinations decreased by 11% relative to 2008Q3. These extensive margin dynamics are not driven by the granularity of varieties. There are similar patterns for firm-destinations and the total number of exporting firms.

These large net entry changes can occur relatively fast given the high churning rates we observe in exporting. For example, between 2003 and 2007, the average gross entry rate for firms exporting in quarter \( t \), but not at \( t - 4 \) was 38% and the gross exit was 34%; in the first year of the crisis (2008Q4–2009Q3) entry fell to 33% and exit jumped to 38%. Moreover, the decline in annual export net entry does not reflect changes in domestic firm births and deaths during the Great Recession. Prior to the crisis, average annual domestic births and deaths were both roughly 10%. In 2009, births fell to 8% and firm deaths increased to 11%.3 One important implication of these high export churning rates is that they permit relatively fast adjustment when conditions worsen, even if exit from a market is mainly due to attrition as assumed by other work (e.g. Ghironi and Melitz, 2005; Bilbiie et al., 2012).

2.3. Aggregate growth decompositions

We decompose aggregate export growth into its intensive and extensive margins to measure the aggregate impact of variety dynamics. We index trade value flows, \( x_{vit} \), by firm-product (\( v \)), destination (\( i \)), time (\( t \)) and type \( z \in \{ \text{ENTRY}, \text{EXIT}, \text{CONT} \} \). The extensive margin is the sum of \( \text{ENTRY} (x_{vit} > 0 \text{ and } x_{vit-4} = 0) \) and \( \text{EXIT} (x_{vit} = 0 \text{ and } x_{vit-4} > 0) \). The intensive margin is comprised of continuers \( (x_{vit}, x_{vit-4} > 0) \). We compute a midpoint growth rate:

\[
\dot{x}_{vit}^z = \frac{x_{vit}^z - x_{vit-4}^z}{\frac{1}{2} [x_{vit}^z + x_{vit-4}^z]} \in [-2, 2].
\]

We let \( f^z = 1 \) if a trade flow belongs to margin \( z \) and write the aggregate export midpoint growth rate as the sum of these mutually exclusive margins

\[
\dot{X}_t = \sum_{i,v} f_{vit}^z x_{vit}^z \times x_{vit}^z.
\]

where the weights, \( s_{vit}^z = \frac{x_{vit}^z + x_{vit-4}^z}{x_{vit}^z + x_{vit-4}^z} \), are the shares of flow \( z \) in average total exports from \( t \) to \( t - 4 \).

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3 These extensive margin figures for exporting reflect the universe of all trade transactions in the LFTTD matched to a firm in the Census Business Register. The domestic birth and death rates are from the Longitudinal Business Database (LBD). Our subsequent aggregate decomposition figures and Table 1 reflect the subsample described in the appendix used for regression analysis in Carballo et al. (2018).
In Fig. 4, we plot the resulting annual growth, by quarter, of the intensive and extensive margins. Both contribute to negative export growth when the GTC began. The decline of the intensive margin was larger, but so was its reversal. The extensive margin dynamics in Fig. 4 are quantitatively important for U.S. export growth. In Table 1, we see that before the GTC, the extensive margin accounted for 36% of export growth on average. During 2008Q4–2009Q3 the extensive margin contributed to the decline is more than 6 points on average, or 25% of the observed aggregate decline. After exports started to grow again, the extensive margin made a positive contribution to growth that was smaller than before the crisis.

The export dynamics were different for PTA destinations, particularly for the extensive margin. In Fig. 5, we decompose the margins of aggregate export growth, $^X z gt = \sum_{v,l} v_{gt} \times \hat{X}_{vl,t}$ from Eq. (2), aggregated up to PTA and non-PTA groups. Before the crisis, the extensive margin contribution was similar for both destinations. But in each quarter of the GTC the non-PTA contribution of the extensive margin was higher relative to PTA destinations in the decline and recovery. The share of export growth in the extensive margin is larger for non-PTAs relative to PTAs throughout the crisis and recovery, as shown in Table 2.

Are these margin differentials predicted by standard determinants? We estimate a gravity regression using pre-crisis data and use it to predict each margin during the crisis. Specifically, we use the intensive and extensive growth margins plotted in Fig. 5 over 2003Q1–2008Q3. We run $\hat{X}_{gt} = \gamma_{pt} PTA + \gamma_{z} \Delta \ln GDP + \gamma_{d} \Delta \ln GDP\text{def} + \alpha_t + \epsilon_{gt}$ where $z$ is the continuing or net entry margin to the aggregate growth of each group $g = \{PTA,\text{NON}\}$ during $t < 2008Q4$. We include a PTA indicator, controls for the export weighted average change in destination GDP and the GDP deflators by group, and time effects, $\alpha_t$.

We predict each margin for the out-of-sample quarters starting in 2008Q4, compute the unexplained residuals and verify if they differ by PTA status. We obtain the predicted value, $\hat{X}_{gt,t}$, based on the realized GDP and deflator growth at $t$ and the estimated coefficients while assuming $\alpha_t = 0$ for $t \geq 2008Q4$. The difference between the data and this prediction is the unexplained residual $r_{gt,t} = \hat{X}_{gt,t} - \hat{X}_{gt,t}$. If the pre-crisis model fits the crisis data well, then the average of residuals in any given period is
$E(r_{zt}^{PTA}) = E(\alpha z^t)$, i.e. the residual for each group would reflect any common unobserved shock; but their differential would not, i.e. $E(r_{zt}^{PTA}) = E(r_{zt}^{NON})$.

In Fig. 6, we plot the unexplained residuals. The left panel shows no significant differential for the intensive margin. The right panel shows the non-PTA extensive margin growth is 4.6 pp below its predicted value on average in the first year of the crisis whereas it is zero for PTAs. The differential PTA dynamics for the extensive margin cannot be explained by standard determinants. We will see how an uncertainty-augmented model can account for this finding.

2.4. Discussion

To summarize, there is an important differential between PTA and non-PTA exports in the GTC. We find that (i) overall export growth toward PTA destinations was relatively higher during the GTC and recovery; (ii) the extensive margin played an important role in the recent evolution of U.S. exports, and accounted for up to a third of annual export growth after the onset of the financial crisis in late 2008; and (iii) there is a larger extensive margin adjustment in non-PTA markets, which is not explained by standard determinants.

Why may we expect these differences even when non-PTA countries are WTO members? First, tariff commitments in the WTO take the form of maximum ceiling rates and several countries’ applied tariffs were below the ceilings and could be raised. In the PTAs we consider, those commitments are typically a zero tariff. Second, agreements are self-enforcing and thus cooperation is...
typically higher in PTAs where monitoring is easier and retaliation incentives are stronger. In contrast, the incentive to deviate and increase protection for a short-run payoff can be large in downturns (Bagwell and Staiger, 1990, 2003).

We now provide a model to better interpret these facts and provide a framework to quantify the impact of economic and policy uncertainty on firm export decisions.

3. Theory

We develop a dynamic model of firm export decisions under multiple uncertainty shocks. We extend Handley and Limão (2015) in different dimensions. First, we allow for uncertainty in export conditions, which capture foreign trade policy, other shocks affecting export profit, and their interaction. Second, we model government preferences for foreign market access under uncertainty. Third, we introduce destination-specific export exit and re-entry, which captures the heterogeneity described above; we also examine the dynamics of exporting more broadly and derive exact relationships that map observable data to sufficient shocks that we use for the quantitative analysis.

3.1. Environment

The operating profit for an incumbent monopolistically competitive firm, \( v \), that exports a differentiated good to country \( i \) is determined as follows. At the start of each period \( t \) a firm observes all relevant information before making its production and pricing decisions. This assumption and the absence of any adjustment costs implies that after entry with a particular technology firms simply maximize operating profits in a market, \( \pi_{ivt} \), period by period. So operating profits are derived similarly to monopolistic competition models in a deterministic setting.

A firm \( v \) faces a standard CES demand with \( \sigma > 1 \) in \( i \) at time \( t \),

\[ q_{it} = \frac{D_{it} \tau_{it}^{\sigma}}{C^0_1 \sigma_{it}} \rightarrow \quad p_{it} = a_p P_{it}^{\sigma} . \]

The consumer price is equal to the producer price, \( p_{iwt} \), times the ad valorem tariff policy factor, \( \tau_{it} \geq 1 \). The demand shifter, \( D_{it} = \alpha Y_{it}(P_{it})^{\sigma - 1} \), is proportional to aggregate income and the fixed share, \( \alpha \), spent on differentiated products; \( P_{it} \) is the CES price aggregator over all domestic and foreign varieties. From the firm’s perspective, the export conditions term, \( a_d = D_{it} \tau_{it}^{\sigma} \), is exogenous and summarizes all payoff relevant information at \( t \).

\[ \text{Fig. 6. Residual aggregate export margins by PTA status. Notes: Difference of observed and predicted margins by PTA status. See the text for details on the estimation.} \]

\[ \text{In presence of scarce enforcement power, the WTO contains provisions that allow its members to legally increase protection above negotiated levels (cf. Hoekman and Kostecki, 2009, Chapter 9). For example, article XIX of the GATT/WTO provides a safeguard allowing governments to increase protection when a domestic industry is harmed by imports. Since PTA partners are often exempted from such safeguards a U.S. firm faces a lower risk of protection in PTA markets.} \]
Labor is the only factor of production. It has constant marginal productivity in a numeraire sector so the wage is normalized to unity. Differentiated goods are produced with a constant marginal cost, characterized by a labor coefficient of $c_v$. At the start of each period firms know the export conditions, their productivity and $\sigma$. They choose prices to maximize operating profits in each period, $\pi_{it} = (p_{it} - c_v)q_{it}$, leading to a standard mark-up rule over cost, $p_v = c_v/\rho$, where $\rho = (\sigma - 1)/\sigma$. Using the optimal price and demand, we obtain the export revenue received by the producer, and the associated operating profit:

$$p_{it}q_{it} = a_q c_v^{1-\sigma} \rho^{\sigma-1}$$  \hfill (4)

$$\pi_{it} = a_q c_v^{1-\sigma} \check{\rho}.$$  \hfill (5)

where $\check{\rho} = (1-\rho)\rho^{\sigma-1}$.

### 3.2. Export dynamics under economic and policy uncertainty

In this environment, we can analyze firm decisions in any given export market separately, so we omit the destination subscript below. Unless otherwise stated, all the variables vary by destination, except for $c_v$. We start by characterizing uncertainty using a generalized demand regime via a mixture of distributions. This approach allows us to parameterize the risk heterogeneity across destinations and explore how PTAs can affect the interactions across multiple sources of risk.

#### 3.2.1. Export investment under uncertainty

To focus on export entry decisions, we make the following assumptions. First, the exporting country is small, so its firms have a negligible impact on the destination market aggregates. This allows us to focus on the direct effects of demand uncertainty on operating profits rather than indirect GE effects. Second, there are zero domestic entry costs and a constant domestic mass of potential firms.\(^5\)

A firm must incur a sunk cost, $K$, to start exporting to a specific market. Given the current conditions, it will be optimal to enter if the expected value of exporting, $\Pi_e$, net of $K$ is at least as high as the expected value of waiting, $\Pi_w$. So the marginal entrant at any given $q_t$ is the firm with costs equal to $c^U_t$, which is a cost cutoff defined implicitly by

$$\Pi_e(q_t, c^U_t, r) - K = \Pi_w(c^U_t, r).$$  \hfill (6)

Before export entry, the firm observes the current conditions, $q_t$. It forms expectations regarding future profits using information about a generalized demand regime $r = \gamma, M$, which the firm takes as given. Firms believe that a demand shock in the following period occurs with probability $\gamma$ and when it does the new demand parameter, $a'$, is drawn from a distribution $M$, independent of the current $a$.

The demand regime has the following characteristics. We have a distribution $M(m_s, H_{\gamma}(a)) = \sum_{m_s} m_s H_{\gamma}(a)$, which is a mixture over $s \geq 1$ exogenous distributions $H_{\gamma}$ for the mutually exclusive combinations of states with fixed mixing weights $m_s \in [0, 1]$ and $\sum m_s = 1$. This characterization has several advantages. First, different regimes can encompass a range of situations, e.g. if $\gamma = 0$, there is no uncertainty; if $\gamma = 1$, then demand is i.i.d. If $\gamma \in (0, 1)$, then it captures demand persistence, i.e. $a$ has serial correlation, and there are imperfectly anticipated shocks of uncertain magnitude. Second, it captures multiple sources of shocks driving $a$ without assuming explicit distributions. Third, by varying the weights across what we refer to as uncertainty states, $s$, we can characterize heterogeneous risk across destinations and isolate the source of that risk (e.g. by shifting probability from a state $s$ with negligible policy risk to another where it is high).

We assume firms have no per period fixed cost after entry, so it is always optimal to export after entry. We allow a firm’s export entry capital to specific markets to fully depreciate with probability $d$, which is independent across markets. The depreciation process is simple: at the end of each period, the export capital either fully depreciates or remains intact. When full depreciation occurs, the firm can re-enter if it decides to make the sunk investment $K$ again. The decision is independent of whether or not it previously exported. This process generates exit from exporting without firm death. It also allows for exit in a subset of markets, whereas if the firm only exited upon death, it would always exit all markets.

How does depreciation affect entry decisions and discount factors? Given this setup, the initial entry decision is independent of whether a firm will ever be able to re-enter that market after re-paying the cost, provided we use an effective discount rate that reflects the probability that the capital survives, given by $1 - d$.\(^6\) This implies that we can solve for market entry as if the firm had only one possibility to enter and had to choose when to do so. Then if the firm’s capital depreciates, it will evaluate the entry decision again unless the whole firm dies (with probability $\check{\delta}$). So the firm’s effective discount rate used to value future export

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\(^5\) Handley and Limão (2017) allow for GE effects of policy uncertainty via the price index. This introduces adjustment dynamics that attenuate, but not overturn, the direct effects of tariff policy on entry decisions.

\(^6\) The intuition should be clear: the re-entry decision of any given firm is independent of its past export status if it has lost all its export capital. There is no other measure of experience or presence in the market that is relevant for exporting; each entry decision can be made independently of future re-entry. We prove this in the NBER working paper (Carballo et al., 2018).
payoffs is \( \beta = (1 - \delta)/(1 - \delta) < 1 \). Since there is a fixed probability of death, \( \delta \) there is an equal probability of new firms being born to replace those that die. This maintains a constant mass of active domestic firms.

One implication of this framework is that while exit rates are exogenous, measured gross exit still depends on current conditions and entry cutoffs. In stationary states, where \( c^0_i \) and entry decisions are unchanged relative to the previous period, the gross exit rate equals the death rate \( \delta \), since the firms that lost their export capital re-enter. The same is true if conditions improve. But in periods where conditions worsened, the measured exit will exceed \( \delta \) since some surviving firms that lost their export capital do not re-enter. The relevant adjustment dynamics are derived in Section 3.4.

We derive solutions for the values of entry \( \Pi_e \) and waiting \( \Pi_w \), solve for the cutoff condition under uncertainty, and prove the propositions in Appendix B. Proposition 1 provides the key expressions for the cost cutoff and how it changes with uncertainty shocks.

**Proposition 1. Uncertainty and entry under multiple shocks**

Under an uncertain demand regime \( r = (\gamma, M(m_s, H_s(a))) \) with volatility \( \gamma \) and conditional probability \( m_s \) for each shock \( s \):

(a) For any value of \( a_s \), the cutoff \( c^0_i \) for the firm that is indifferent between entry or waiting at time \( t \) is

\[
c^0_i = c^0 \times U_t = \left[ \frac{a_t \hat{\sigma}}{(1 - \beta) R} \right]^{\frac{1}{\gamma - 1}} \times \left[ 1 + \frac{3\gamma(\beta - 1)}{1 - (1 - \gamma)} \right]^{\frac{1}{\gamma - 1}}
\]

(b) The cutoff in eq. (7) is decreasing in \( \gamma \).: \( \frac{\partial \ln c^0_i}{\partial \gamma} \leq 0 \) with a strict inequality for all \( a_t > a_{\text{min}} \).

(c) Shifts towards any given riskier shock, \( \Delta m_s = -\Delta m_t > 0 \), where \( H_s \) SSD \( H_r \) increase overall demand uncertainty and lower entry:

\[
c^0_i(\gamma, M(m_t + \Delta m_s, H_s)) < c^0_i(\gamma, M(m_t, H_s)).
\]

(d) The entry effect in (c) is magnified by demand volatility:

\[
\frac{\partial \ln c^0_i(\gamma M(m_t, H_s))}{\partial \gamma} \geq \frac{\partial \ln c^0_i(\gamma M(m_t, H_r))}{\partial \gamma}
\]

Part (a) establishes a cutoff \( c^0_i \) for the firm that is indifferent between entry or waiting at \( t \) for any \( a_s \). The cutoff under uncertainty is lower than the deterministic cutoff, \( c^0_i \), whenever the uncertainty factor, \( U_t \), is less than unity. This occurs if and only if \( \hat{\sigma}(a_t) \leq 0 \). The latter measures the average tail risk to operating profits conditional on a shock. Each of the tail risk terms in (9) represents the expected loss in operating profits times the probability \( H_s(a_s) \) of a shock that worsens conditions in state \( s \).

In part (b) we show that volatility lowers entry. We start with a simplification of the shock process and then generalize in two dimensions. Consider increases in \( \gamma \) at a given \( H_s \), i.e. let \( m_s = 1 \), and assume variation in \( a_s \) is only from trade policy. Increasing \( \gamma \) makes both higher and lower demand levels more likely and this lowers the expected value of exporting, \( \Pi_e \), relative to \( \Pi_w \) in (6). To see why, we note that probability of a higher demand increases both \( \Pi_e \) and \( \Pi_w \) in (6) by the same amount for a marginal entrant; whereas lower demand reduces \( \Pi_e \) but not \( \Pi_w \) because the firm will not enter if conditions worsen. This captures what Bernanke (1983) described as the “bad news principle”.

We generalize this simple case to handle (1) different distributions of demand shock from any source and (2) shifts in the riskiness of the distribution of demand. First, we allow \( a_s \) to reflect a general demand shock (e.g. policy and economic conditions) and let the demand regime be described by \( \gamma \) and the joint distribution of shocks, \( H(a) \). We can then compare cutoffs in two regimes where \( H \) SSD \( H_r \), holding \( \gamma \) fixed. The riskier distribution has thicker tails and thus implies larger losses conditional on a bad shock. The cutoff is lower for the riskier distribution, \( c^0_i(H) \leq c^0_i(H_r) \) because \( \hat{\sigma}(a_t) \leq \hat{\sigma}(a_t) \). Second, within each state, \( c^0_i \) is the expectation over \( H_s \). So \( \hat{\sigma}(a_t) \) in (8) is the average of these expected losses weighted by probability \( m_s \) of each state. By generalizing to mixing weights \( m_s \in [0, 1] \) we can consider shocks to risk, which we will use later to consider government preferences and to differentiate between regimes.

Part (c) establishes the impact of increasing risk at a given \( \gamma \). Consider a change in the probability of the shock \( s \) such that \( \Delta m_s = -\Delta m_t \), where \( H_s \) SSD \( H_r \). From Eq. (7), we see that this change only affects the proportional loss term and thus \( c^0_i(H) \leq c^0_i(M) \) if and only if it implies \( \hat{\sigma}(a_t) \leq \hat{\sigma}(a_t) \). In the appendix, we show that this condition holds for any \( a_t \) if and only if \( H_s \) SSD \( H_r \): the latter has thicker tails and thus implies larger losses conditional on a bad shock. Part (d) establishes that the reduction in entry due to higher risk in part (c) is magnified by increases in volatility, \( \gamma \).

The results in Proposition 1 do not require additional restrictions on the long-run mean of \( a \) across the regimes. For example, part (b) still holds if the initial \( a_t \) is at its long run mean (so that shocks to \( \gamma \) leave the mean unchanged). Parts (c) and (d) hold for any case where \( H_s \) SSD \( H_r \), including the special case of a mean preserving spread. We illustrate these points in Fig. 7(a) where \( H_s \) SSD \( H_r \) correspond to the extreme cases where \( m_s = 1 \) and \( m_r = 0 \), respectively. Any other weights represent intermediate risk,
e.g. $m_s = 1/2$. We normalize the distributions such that $E(a) = 1$ to focus on mean preserving spreads of $a$. Panel (b) of Fig. 7 shows the impact on the cutoff of increasing volatility from zero to the maximum at any $a_t$ (i.e. $\ln U_t = \ln [c^H(a_t, \gamma = 1) / c^D(a_t, \gamma = 0)]$). At every $a_t > 0$, we can see that the riskier market has the largest cutoff reduction under uncertainty, as shown in Proposition 1(c).
3.2.2. Role of sunk costs and endogenous timing

We highlight the role of sunk costs and endogenous timing for Proposition 1. A simple corollary of Proposition 1 is that if export costs are not sunk, then firm export entry and sales at any $a_t$ are independent of demand uncertainty. The intuition is simple: in this model, if costs are not sunk, then entry affects only current profits. This alternative can be nested as a special case of our model and entails an entry cutoff equivalent to the deterministic cutoff in Proposition 1, i.e. $c_t = c_D^t$. Therefore, the relative cutoff in our model relative to an alternative without sunk costs is $c_U^t/c_D^t = U_t$, i.e. it is fully captured by the uncertainty factor. Hence, the results in parts (b), (c) and (d) also characterize how uncertainty lowers entry relative to a model without sunk costs.

Waiting is only valuable if firms can enter at a time when they can take advantage of improved conditions. If firms cannot choose this timing, then we get different entry results. For example, if firms must make a now or never entry decision, then they do so if and only if the sunk cost is lower than the expected value of export profits, $I_t$. This value is derived in the appendix and reflects the current and future expected profits, $π(a_t)$ and $E_t[π(a_t)]$ respectively. Since profits are linear in $a_t$, the entry decision is increasing in the mean of $a$ and no other moments, so a MPS would not affect entry. Moreover, the main effect of increasing $γ$ is to increase the probability of leaving the current conditions, so it lowers entry if and only if those conditions are below the mean, $a_t < E_t(a_t)$.

3.2.3. Aggregate trade value and elasticity

We derive some implications of Proposition 1 for aggregate exports, which will be useful for the quantitative analysis. Uncertainty lowers aggregate exports and attenuates its elasticity with respect to $a$ due to the entry effects in Proposition 1. The reduction in exports is caused by fewer exporting firms at any $a_t$. The elasticity attenuation reflects a caution effect arising from the probability that an improvement in $a_t$ may be reversed. We can isolate the caution effect by focusing on stationary period exports: when no exporting firms have costs above the current export entry cutoff. We denote stationary exports by $R(\mathcal{a}_t,c_U^t(a_t))$, which separates the extensive margin effects via $c_U^t$, and note it is given by

$$R(\mathcal{a}_t,c_U^t(a_t)) = a_t n \rho^{a_t-1} \int_0^{c_U^t(a_t)} e^{\sigma(a_t)} dF(c),$$

(10)

where $F$ is the cost distribution and $n$ the exogenous mass of potential exporters. The trade elasticity with respect to export conditions is

$$\varepsilon(a_t) = \frac{d \ln R(\mathcal{a}_t,c_U^t(a_t))}{d \ln a_t} = 1 + k(c_t) \left( 1 - \frac{1}{\sigma - 1} \right) + \frac{\partial \ln U_t}{\partial \ln a_t} > 0, \tag{11}$$

which is the sum of the unit intensive margin elasticity and the extensive margin component; $k(c_t) = \frac{\partial \ln R(a_t,c_U^t(a_t))}{\partial \ln a_t} > 0$ defines the export entry elasticity.

Uncertainty has two interesting implications for this trade elasticity. First, there is now a caution effect because increasing $\rho$ raises tail risk—this attenuates but does not overturn the standard entry effect of increasing $\rho$ so $\varepsilon(a_t) > 0$ (see Appendix B.2.2). Second, the caution effect depends on current conditions and implies a variable elasticity under uncertainty even if $\varepsilon(a_t)$ is constant under certainty. For example, under the standard untruncated Pareto productivity with dispersion $k$ we obtain $k(c_t) = k - (\sigma - 1)$ and a familiar elasticity $\varepsilon(a_t) = k/(\sigma - 1)$ under certainty.

This trade elasticity points to the importance of allowing for uncertainty shocks to rationalize the larger extensive to intensive movements for non-PTAs found for the GTC. Without uncertainty shocks, those relative movements would require shocks to $a_t$ and their effect on $\ln R$ from the extensive relative to intensive margin is $\frac{\partial \ln R(a_t,c_U^t(a_t))}{\partial \ln a_t} \sigma(c_U^t) / \partial \ln a_t = \varepsilon(a_t) - 1$. Under certainty and constant $k(c_t)$ (or starting at the same $a_t$) this ratio is constant and identical across destinations of U.S. firms. To generate the observed pattern of extensive/intensive effects from $a_t$ alone would require $\varepsilon(a_t)$ to vary systematically across countries, e.g. due to caution. However, in the quantitative section, we will see that such changes do not explain the observed margins pattern.

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7 We thank a referee for suggesting we clarify these.
8 To see this denote the period entry capital by $K$ and assume it fully depreciates each period, implying $β_t = 0$. Setting current export profit equal to the entry cost we obtain $c_t = |a_t|/K$, which is the same as $c_D^t$ when we impose the sunk cost to be equivalent to the PDV of period costs so $K = (1 - β_t)K$.
9 This expression applies if the cutoff exceeds the historical maximum such that $c_U^t > \max_{c_D^t} c_D^t$, i.e. entry is currently easier than ever before. Otherwise, we must account for the legacy of surviving exporters. These are firms that started exporting under better conditions and remain since operating profits are positive once the sunk cost is paid. This generates an additional source of attenuation but only after negative shocks. This attenuation due to sunk cost hysteresis is well understood and it is also present in models without uncertainty.
10 Key results on gains from trade rely on constant trade elasticities and recent research highlights the importance of allowing for variable elasticities, e.g. via productivity distributions other than untruncated Pareto (cf. Melitz and Redding, 2015), or variable markups (Arkolakis et al., 2018). Uncertainty generates variable elasticities even with Pareto, CES and monopolistic competition.
11 We use this definition of intensive vs. extensive to make the point clearly using the expression in Eq. (10) but, in the quantitative section, we will use the midpoint margins as defined in the stylized facts.
Proposition 1(c) predicts that a shift to a riskier distribution reduces entry. In the quantitative section we will see the model can rationalize the data if non-PTA destinations faced a larger increase in the probability of a riskier distribution, $\Delta m_{t}$, than the PTAs. This occurs at the start of the crisis and is then reversed.

3.3. Agreements, endogenous uncertainty, and trade

We provide a government objective that generates a motive for trade agreements that reduce demand uncertainty and its impacts on average export outcomes. We first assume the agreement is unrestricted, so it can address any demand uncertainty reflected in $a$. This benchmark is useful to establish basic results without additional structure on the source and interaction of shocks in $a$, but it applies only to agreements where trade policy is either (i) the only source of uncertainty or (ii) contingent on any shock affecting $a$. We then extend the results to agreements where neither conditions (i) or (ii) hold, which is more realistic and essential to derive the conditions when economic shocks magnify policy uncertainty. Given our subsequent application we refer to preferential trade agreements (PTAs), but the results apply to any agreement with a small enough exporter.

3.3.1. Government objective and agreement motive

PTAs internalize the costs of certain policies on foreign exporters. A government has a PTA motive if some change in the foreign policy parameters faced by its exporters improves its objective function relative to the non-PTA case, i.e. $G^{PTA} > G^{M}$. Most PTA models are deterministic so governments choose an initial policy level, which remains in place indefinitely. We denote a change in policy level due to the PTA at time $t$ by $\Delta^{PTA}$. Since there are time-varying incentives for governments to set protection, we must also specify how a PTA affects future policy.

Under an unrestricted PTA, governments can choose from alternative demand distributions. Specifically, we model unrestricted PTAs as a choice of the belief parameter, $m_{a}$, for any state $a$ with a different distribution of $a$. For simplicity, we consider only two states: $s'$ with probability $m$ and $s$ with probability $1 - m$. We define $s$ as the state with lower overall demand risk so that $H_{s}$ SSD $H_{s'}$. We assume PTAs are unable to affect the arrival of any demand shock, so $\gamma$ is taken as given.

In sum, an unrestricted PTA can change two parameters: the belief about the probability of shocks, $\Delta^{PTA}_{ma} = m^{PTA} - m$, and some current policy level that determines current export conditions, $\Delta^{PTA}_{a}$. In the context of trade negotiations, market access improvements correspond to changes in policies that increase export sales (and thus profits). Foreign policy affects exports only via $a$ so we write the reduced form government objective as

$$G = G(a_{t}, M(a), \gamma)$$

and state that the government values market access and is export risk averse if (i) $G_{a_{t}} \geq 0$ and (ii) $G(a_{t}, M(a), \gamma) \geq G(a_{t}, M'(a), \gamma)$ for all $a_{t}$ whenever $M$ SSD $M'$ (with equality at $\gamma = 0$).

The partial effect of $a_{t}$ on $G$ in condition (i) holds in standard policy models without uncertainty; in these models the condition $G_{a_{t}}|_{\gamma=0} \geq 0$ typically reflects the social or political weight given by a government to a measure of aggregate export profits and provides a market access motive for a PTA. We assume this continues to hold under uncertainty, but note that $G_{a_{t}}$ may now be attenuated since it reflects improvements in current market from temporary policies that change with probability $\gamma$. Condition (ii) is a definition of export risk aversion when $a$ affects $G$ only through the export channel and provides what we define as an insurance motive for a PTA.\footnote{Both conditions hold at any given $\gamma$ since we assume the agreement does not affect it. But demand volatility clearly affects the agreement. If $\gamma = 0$ permanently, then there would be no motive for the agreement to address risk. We assume that governments treat $\gamma$ as a fixed parameter (as firms do) so the agreement reflects the level of $\gamma$ when signed.

Our main goal in this section is to define the scope of alternative agreements, i.e. the policy levels and distributions they can affect, and provide broad conditions for a government objective to satisfy in order to gain from an agreement that reduces demand uncertainty. Future research can explore the interesting question of what specific models satisfy the conditions. We further discuss this issue in the online appendix.

The stationary effects capture outcomes after any legacy firms have exited, so they isolate uncertainty motives that are distinct from those caused by sunk cost hysteresis that are present in models without uncertainty. In the working paper we derive the effects of a PTA that also changes initial market access.}

The insurance motive is present in certain models where $G$ is increasing in aggregate export values or profits. Two special cases illustrate this. If the government is impatient and values only exports in the first period of implementation, then $G$ could be any increasing function of aggregate exports, since $R(a_{t}, M)$ is decreasing in risk, as Proposition 2 will show. Second, suppose the government values only changes in long-run outcomes, so the gain from the PTA is $\mathbb{E}_{M}g(a_{t}, M) - \mathbb{E}_{M}g(a_{t}, M')$, then we need $g$ to be some concave function of $R$. Proposition 2 shows that $g = \ln R$ would generally suffice.\footnote{The stationary effects capture outcomes after any legacy firms have exited, so they isolate uncertainty motives that are distinct from those caused by sunk cost hysteresis that are present in models without uncertainty. In the working paper we derive the effects of a PTA that also changes initial market access.}

The reduced form objective in (12) is sufficient to establish when an exporter government has a motive for a PTA and the government’s desired changes in current market access and risk. The following Proposition focuses on a PTA that addresses the insurance motive and its impact on stationary exporting firms and values at any $a_{t}$.\footnote{Both conditions hold at any given $\gamma$ since we assume the agreement does not affect it. But demand volatility clearly affects the agreement. If $\gamma = 0$ permanently, then there would be no motive for the agreement to address risk. We assume that governments treat $\gamma$ as a fixed parameter (as firms do) so the agreement reflects the level of $\gamma$ when signed.} To also derive the average effects over all $a_{t}$, we require two related entry elasticities to be non-increasing. We define the entry elasticity as $k(c) = d \ln F(c)/d \ln c$ and require $dk(c)/dc \leq 0$; we use an analogous condition for the export entry elasticity previously defined, $dk(c)/dc \leq 0$. In appendix B we show they are satisfied by several standard cost distributions.
Proposition 2. Agreements, endogenous uncertainty and export impacts

If a government is export risk averse, then it has an insurance motive for a PTA and it gains from an agreement with a distribution, $M(a)$, that SSD the original, $M’(a)$. These PTAs

(a) increase the stationary number of exporters and export value at any $a_t > a_{\text{min}}$;
(b) increase the expected log of stationary
- number of exporters, $\mathbb{E}_M \ln F(c^U(a,M)) > \mathbb{E}_{M’} \ln F(c^U(a,M’))$, if the entry elasticity is non-increasing;
- export value, $\mathbb{E}_M \ln R(a, c^U(a,M)) > \mathbb{E}_{M’} \ln R(a, c^U(a,M’))$, if the export entry elasticity is non-increasing.

The definition of export risk aversion implies a gain from reducing demand risk, which a PTA can achieve by shifting weight towards the less risky state. The Proposition then derives the impact of PTAs that do so. The entry effect in part (a) is a corollary of Proposition 1(c), which also implies an increase in aggregate export values at any $a_t$. Export profits also rise (since they are increasing in export values) so if $G$ places enough weight on aggregate export profits (or values), then it is better off under this PTA at any given $a_t$.

Part (b) establishes the effects on expected values. We focus on two standard measures used in empirical work, log number of export firms and values, and derive implications for their average across stationary equilibria. The difference between the PTA and non-PTA average is decomposable into a cutoff and a risk compression term. For the number of exporters, we have

\begin{equation}
\mathbb{E}_M \ln F(c^U(a,M)) - \mathbb{E}_{M’} \ln F(c^U(a,M’)) = \int_a \ln \frac{F(c^U(a,M))}{F(c^U(a,M'))} dM’ + \int_a \ln F(c^U(a,M)) d(M - M’).
\end{equation}

The cutoff effect is positive at any $a_t$ as shown in part (a), and thus so is its average over any given distribution of $a$ and of $t$. The term we label as “compression” reflects the change in probability of each $a_t$, which is positive if $F$ is log-concave in $a_t$ (since $M$ SSD $M’$).

We then prove that a non-increasing entry elasticity is sufficient to ensure $F$ is log-concave in $a_t$. This occurs because in our model $c^U(a_t)$ is log-concave in $a_t$ as we prove in the appendix. The intuition is simple if we consider a constant elasticity distribution; in this case, the only impact of the compression in $a_t$ due to the PTA is to increase both the average in $c_t$ (given it is log concave) and thus the average log number of exporters. The effect is reinforced if the elasticity is decreasing (and partially offset otherwise). Several common distributions have non-increasing elasticity, including the power function we use in our parameterization.

A similar decomposition and explanation applies to export values. We prove that a non-increasing export entry elasticity is sufficient for exports to be log-concave in $a_t$ and thus implies a positive compression effect. Intuitively, $R$ is log-concave in $a_t$ for fixed entry, and this can only be offset if the entry elasticity is increasing sufficiently fast. Several distributions imply a non-increasing export entry elasticity. \cite{15}

This Proposition has three simple corollaries. First, even if the firm cost distribution implies $\ln R$ or $\ln F$ is linear in $a_t$ the PTA would still imply a higher average in our model due to the cutoff effect, but not in the alternative one without sunk costs—the latter model only has compression effects. Second, under the conditions in 2(b) a PTA that reduces risk would also benefit any government that values increases in the expected value of (log) exports or number of firms. Third, any increase in risk in $M’(a)$ (e.g. the start of the trade collapse) has a smaller effect under the PTA because it places a lower probability on it.

3.3.2. Restricted agreements and multiplicative shocks

Agreements generally address only a subset of shocks such as policies affecting export conditions. We extend the model to characterize which restricted agreements can achieve the reduction in export risk required by Proposition 2. We also show how other economic risks and their correlation with policy risk affect this restricted agreement.

To analyze the sources of export risk, we rewrite export conditions as a product of components, $a = \prod x$, which we subsequently interpret. Each period there is a probability $\gamma$ of a draw for all variable components from a time-invariant joint density $h_t(x)$. The CDF of export conditions, $H_t(a)$, can then be written as a function of $h(x)$. We define the unrestricted agreement as the one where a government can choose the weights $m_x$ to place on any $x \in S$, where $S$ is the set of all possible joint distributions. In a restricted agreement, the government can only place weight on a subset $\mathcal{S}$ of distributions capturing the subset of shocks the agreement covers.

What type of restricted agreements can lower export risk as required by Proposition 2? To answer this, we impose additional structure on the type and distribution of shocks. We partition the shocks into an economic and trade policy component, $x = \{D, f\}$ by recalling that $a_t = D \times f$ where $D$ is a demand shifter and $f = \tau - a$ is the relative demand for exports, or freeness of exports when facing a tariff $\tau$. We focus on restricted agreements that can only target $f$ by choosing from alternative marginal densities for this variable, $h_t(f)$. The agreement takes the marginal density of the economic shocks summarized in $D$ as given. The latter include shocks to technology, which affect aggregate income, and to preferences, which affect the share of expenditure on differentiated goods. \cite{16}

\[ \text{Recall the demand shifter is } D_t = a_t Y_t (P_t)^{\alpha} \in \mathbb{R} \text{ so technology shocks can affect aggregate income } Y \text{ and demand shocks change the expenditure share on differentiated goods, } \alpha. \text{ Other shocks can affect the price index of differentiated goods } P. \text{ Under this approach we can ensure that changes in the distribution of } f \text{ do not affect } P \text{ if we assume the exporter is sufficiently small, as we have in the derivation of the cutoffs under uncertainty.} \]
To derive clear theoretical predictions we focus on lognormal distributions. This ensures non-negative values for each \( x \), and thus for \( a \) and provides a parametric ranking of distributions according to SSD described below. Specifically, the multiplicative shocks are drawn from a bivariate log normal with correlation parameter \( \eta \) so we have:

\[
\begin{align*}
    x & \sim \ln N(\mu_x, \Sigma_x) \\
    a & \sim \ln N(\mu, \Sigma), \quad \mu = \mu_0 + \mu_f; \Sigma = \Sigma_0 + \Sigma_f^2 + 2\eta \Sigma_f \Sigma_D.
\end{align*}
\]

The first line represents the marginal distribution for each \( x \), which is lognormal so \( (\mu_x, \Sigma_x) \) are the mean and standard deviation for \( \ln x \). Thus, in a restricted agreement the government chooses over a subset \( s \in S \) that includes the possible set of policy parameters \((\mu_0, \Sigma_0)\) with each combination leading to a different \( H_s(a) \).

We use the following result to rank risk states.

**Lemma: SSD ranking conditions** (Levy, 1973): When \( a \sim \ln N(\mu, \Sigma) \) we have \( H_s(a) \) SSD \( H_{s'}(a) \) iff (1) \( \Sigma \leq \Sigma' \); (2) \( \mu \geq \mu' \); and (3) \( \mu + \Sigma^2/2 \geq \mu' + \Sigma'^2/2 \) with either (1) and/or (2) strict.

Conditions (1) and (2) are standard if we were simply ranking a normal distribution, e.g. \( \ln a \); but a lognormal also requires (3) to ensure \( E_s(a) \geq E_{s'}(a) \).

Fig. 8 shows the ranking of distributions with different parameters. The red curve represents condition (3): the iso-arithmetic mean where \( E_s(a) = \text{exp} \left( \mu + \Sigma^2/2 \right) = 1. \) The box labelled \( H_s \) is the combination of parameters used in Fig. 7 and any \( H_s \) along the iso-mean with \( \Sigma' > \Sigma \) represents a mean-preserving spread, e.g. the diamond marks the distribution \( H_{s'} \) in Fig. 7. More generally, \( H_s \) SSD any \( H_{s'} \) in the red area below the iso-mean curve where \( \Sigma \leq \Sigma' \). Any distribution in the blue area above the iso-mean with a parameter lower than \( \Sigma \) will SSD \( H_s \); so any restricted agreement that changes \( (\mu, \Sigma) \) into that region reduces export risk. The remaining ones cannot be ordered.\(^{17} \) Recall that the MPS notion of risk is a special case of SSD that only ranks distributions along the iso-mean line. The theory requires only SSD and thus allows for a ranking of a broader set of distributions and agreements.

What are the features of the restricted agreements that lower export risk? We denote the desired change in export risk identified in Proposition 2 by \( \Delta f = \mu - \mu'_s \), with a similar definition for the change in variance of \( \ln a \): \( \Delta \Sigma^2 \). Recall that the restricted agreement cannot affect the economic parameters, \((\mu_0, \Sigma_0)\), so using (14) and the ranking lemma we obtain the following required changes in the policy distribution. First, \( \Delta \mu = \Delta \mu_s / 2 \), so lowering export risk requires a policy distribution with an increase in the mean of \( \ln f \) equal to that of \( \ln a \). Since \( \ln f = -\sigma \ln \tau \) this agreement is required to lower the mean \( \ln \tau \), as most do in practice. Condition (3) will then require that \( \Delta \Sigma_f / 2 \geq 0 \), with equality in the special case of mean preserving changes in \( a \).

The required change in \( \Sigma_f \) depends on the correlation between economic and policy shocks, \( \eta \). First, if there is no correlation, then we require \( \Delta \Sigma_f^2 = -\Delta \Sigma^2 / 2 \leq 0 \), i.e. the variance of \( \ln f \) must fall by the same amount as that of \( \ln a \). Second, if the correlation is not too negative, then \( \Delta \Sigma_f^2 \leq 0 \). Thus, the restricted agreement must lower the variance of \( \ln f \) and of \( \ln \tau \), as most agreements do in practice.\(^{18} \)

The existing evidence suggests that it is more plausible that \( \eta > 0 \), which amplifies any reduction of \( \Sigma_f \) on \( \Sigma^2 \). In this case, the standard deviation of \( \ln f \) (and \( \ln \tau \)) does not need to fall as much as when \( \eta = 0 \) to achieve a given reduction in export risk. Moreover, when \( \eta > 0 \) the reduction in \( \Sigma_f \) is amplified by higher variance in economic shocks, \( \Sigma_D \). An agreement that achieves the same export risk reduction while requiring smaller changes in the importer policy distribution relative to the status quo is likely easier to reach and enforce and thus more valuable.\(^{19} \)

In sum, there are restricted agreements that lower overall export risk. These agreements are characterized by a freeness of trade measure, \( \ln f \), with higher mean and lower standard deviation (for most correlations), which is consistent with actual agreements.\(^{20} \) Moreover, we argued there is a potential extra benefit of restricted agreements with countries where economic risks are important and shocks to variables such as income and trade openness are positively correlated. Some insights may generalize beyond the lognormal, but this distribution clarifies the key points and will be useful in the quantitative application.

### 3.4. Adjustment dynamics

We now derive the adjustment dynamics triggered by negative shocks in the presence of sunk costs. We also provide an exact decomposition of exports into intensive and extensive margin adjustments that are central in understanding the aggregate implications of the model, as we show in Section 4.

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\(^{17} \) Only those along the vertical line can be ranked relative to \( H_s \) in the FOSD sense.

\(^{18} \) If \( \Sigma_f \) is sufficiently large relative to \( \Sigma \) then there is some \( \eta \in (-1, 0) \) such that increases in \( \Sigma_f \) actually lower \( \Delta \Sigma^2 \).

\(^{19} \) Possible microfoundations for this include costs of contracting, monitoring or enforcing that are increasing in the deviation from the status quo or if the initial parameters resulted from unilateral optimal choices for the importer.

\(^{20} \) Actual agreements do not require that changes in the marginal policy distribution for \( f \) in (14) satisfy the formal reduction in risk in the lemma. But in our working paper we show that any decrease in export risk requires a decrease in policy risk if \( \eta \geq 0 \). Moreover, we show that no increase in policy risk can decrease export risk for any \( \eta \).
The following expressions—for the number of firms and aggregate exports respectively—allow for dynamic effects from shocks that lower the cost cutoff, e.g. worse export conditions (via $\alpha$) or higher uncertainty (via $\gamma$ or $m$).

\[ N_t = nF(c_t^U) + \lambda_t^{nh} \]
\[ R_t = R(a_t, c_t^U) + \lambda_t^{nh} \]

The $\lambda$ terms measure the contribution of legacy firms specific to each outcome. If the current cutoff is above previous levels, then all firms in the market have cost below that cutoff. If the cutoff is lower than a previous period, then there are legacy firms that entered when conditions were better but continue to export until their capital depreciates. Therefore, the legacy terms depend on the history of shocks.

We focus on two mutually exclusive histories denoted by indicator functions $1^h = 1$. Expansions with cutoffs that are higher than any previous cutoff are denoted by $h = 0$. Initial declines or partial recoveries where the cutoff is below the initial one, $c_{t0}^U$, are denoted by $h = +$. For exports, we have

\[
\lambda_t^{nh} = \begin{cases} 
0 & \text{if } 1^h = 1 : c_t^U \geq \max \{c_{s-1}^U, c_{s-1}^L\} \\
\beta \int_{c_{t0}^U}^{c_t^U} \left(a_t, \rho^\alpha - 1, c_t^L - \gamma\right)^{1-\alpha} dF(c) & \text{if } 1^h = 1 : c_t^U \in \left( \max \{c_{s-1}^U, c_{s-1}^L\}, c_{t0}^U \right). 
\end{cases}
\]

The second line captures the fraction of surviving firms with export specific capital at $t$ and costs between the current lower cutoff and the original one. A similar expression applies for $\lambda_t^{nh}$ after replacing the firm export expression in with unity. Denoting the ratio of a variable at $t$ to its stationary period value using a “∼” we obtain the following expressions:

\[
\tilde{N}_t = \tilde{F}_t + \beta^t \left( 1 - \tilde{F}_t \right) \\
\tilde{R}_t = \tilde{a}_t \left[ \tilde{C}_t + \beta^t \left( 1 - \tilde{C}_t \right) \right] \text{ if } 1^h_t = 1.
\]

The ratio of the number of firms in the first line is a function of the change in the cutoff, $\tilde{F}_t = F(c_t^U)/F(c_{t0}^U)$, and, when the shock is negative, it includes the legacy term that decays at a rate $\beta$. The change in exports is a function of shocks to two sufficient statistics: export conditions, $a_t$, and “aggregate” cutoffs $C_t = \int_0^{c_t^U} c_t^{-\alpha} dF(c)$.
4. Aggregate implications of GTC

We use the expressions in the previous section to extract aggregate shocks to export conditions, cutoffs, and thus uncertainty in the GTC. Using these, we determine (i) if the model accounts for the dynamics of the different aggregate margins documented in Section 2 and if sunk costs and uncertainty play a role; (ii) if there are uncertainty shocks; (iii) what type of uncertainty shocks are consistent with the data and whether they are higher for non-PTAs; and (iv) the export dynamics under alternative counterfactual uncertainty shocks. Finally, we ask whether the contribution of uncertainty shocks implied by the model can account for the unexplained extensive margin residuals from Section 2.

4.1. Identification of shocks and model fit

Given data on (i) changes in exports and varieties relative to a pre-GTC period; (ii) a productivity distribution (Pareto); and (iii) parameter values \( \{k, \sigma, \beta\} \) we can solve for shocks \( \{\bar{a}_t, \bar{c}_t^U\} \) in \( t = Q408 - Q411 \). We do so separately for the aggregates of PTA and non-PTA countries. For periods of initial decline or partial recovery we employ (18) as is, otherwise we remove the respective legacy term. We do not require any assumptions on any uncertainty parameters to recover these shocks. We summarize the parameters used and their justification in Appendix C.1 and provide robustness to them before the conclusion.

In the upper panel of Fig. 9 we plot each shock relative to its respective pre-GTC quarter, e.g. \( \bar{c}_{t210} = c_{t210}^U \bar{c}_{t208} \). In the left panel we see that PTA countries experienced a decline in the cutoff in the first year, a return to pre-GTC levels by Q409 and a subsequent stabilization. In the first crisis period, the cutoff for non-PTA, on the right panel, declines by more than for PTA; this occurs even though the export conditions were relatively better for non-PTAs than PTAs (orange dashed line). This is only possible if the implied uncertainty factor, \( U \), falls by more for non-PTAs, since \( \bar{c}_t^U = a_t^U \bar{U}_t \), as shown by the purple line falling below unity in the first period.

The changing \( \bar{U}_t \) has two implications. First, since \( c_t^U / c_t^D = U_t \) the model nests the alternative case of no uncertainty (or no sunk costs); that alternative model entails \( U_t = 1 \), which is rejected by the data. Second, if uncertainty is present but its parameters are constant, then the reduction in \( a_t \) lowers tail risk and that by itself would increase \( U_t \) in the first year, rather than lower it as observed for non-PTAs. More generally, if uncertainty parameters were constant, then \( U_t \) and \( a_t \) would always be negatively correlated (the caution effect derived in Section 3.2.3), which is not true in the first year. To quantify the magnitude of the uncertainty shocks that can rationalize \( U_t \), we will subsequently impose a distribution for \( a_t \).

The model can exactly decompose the dynamics of aggregate exports and varieties with only two types of shocks. But how well can it explain each separate aggregate export margin we document in Section 2? To assess this we derive the intensive and extensive margin as functions of the extracted shocks and data. In the initial quarters of the GTC these yearly growth rates are given below. In the appendix we provide expressions for the remaining periods.

\[
\begin{align*}
R_t^{mid} = & \frac{2}{R_t + 1} \left[ \frac{\bar{a}_t \bar{C}_t}{\text{Entry}} - \left( \delta + (1 - \delta) d (1 - \bar{C}_t) \right) + (1 - \delta) (\bar{a}_t - 1) \left( \bar{C}_t + (1 - d) (1 - \bar{C}_t) \right) \right], \quad 1^T = 1 \tag{19}
\end{align*}
\]

Note that \( R_t^{mid} \) corresponds to the midpoint growth rate as defined by the data in Section 2, so it is a yearly growth rate. The first term on the RHS represents the expression in the model capturing the contribution from entrants, \( \frac{2\bar{a}_t \bar{C}_t}{R_t + 1} \), expressed as a function of the extracted shocks. It reflects the fraction \( \delta \) of firms below the current cutoff that enter to replace those that died. Similarly, the contribution from exit includes the \( \delta \) that die and the \( (1 - \delta) d \) that survive but lose their export capital and do not reinvest in that period because their cost is above the cutoff. The continuing contribution includes all those that survive and are either below the cutoff or above it but did not lose their export capital.

The margins depend on the shocks we recovered and on two additional parameters. We do not allow these parameters to vary over time or PTA status, nor do we calibrate them to explain the trade data. Instead we employ U.S. data on sunk costs); that alternative model entails \( U_t = 1 \), which is rejected by the data. Second, if uncertainty is present but its parameters are constant, then \( U_t \) and \( a_t \) would always be negatively correlated (the caution effect derived in Section 3.2.3), which is not true in the first year. To quantify the magnitude of the uncertainty shocks that can rationalize \( U_t \), we will subsequently impose a distribution for \( a_t \).

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The margins depend on the shocks we recovered and on two additional parameters. We do not allow these parameters to vary over time or PTA status, nor do we calibrate them to explain the trade data. Instead we employ U.S. data on firm and variety exit as described in appendix C.1, these imply \( \delta = .09 \) and \( d = .16 \).

The model can replicate the margins in the data even when we take these parameters as externally given. In the lower panel of Fig. 9 the orange and purple solid lines represent the intensive and extensive margins respectively computed using the model

---

21 Under Pareto we have \( \bar{C}_t = (\bar{a}_t)^{1/(\alpha - 1)} \). The values for \( k \) and \( \alpha \) are in the range of those typically used; the effective export discount \( \beta \) is consistent with firm-product exit rates and capital depreciation discussed in the appendix.

22 We assume \( t = Q407 - Q308 \) are stationary, so \( 1^T_t = 1 \), which is consistent with the trend of expanding firm-products leading up to those periods. For GTC periods, if \( \bar{N}_t \geq 1 \) then we also have \( 1^T_t = 1 \), this is true for \( t = Q410 - Q311 \). To determine if the history of other GTC periods has \( 1^T_t = 1 \) we first check the necessary condition that \( \bar{N}_t < 1 \). This condition is sufficient for \( t = Q408 - Q309 \) since their respective value in the previous year were stationary (so a decline). To apply (18) to \( t = Q409 - Q310 \) we must verify not only that \( \bar{N}_t < 1 \) (relative to pre-GTC) but also that the implied cutoff in each of those quarters is above the respective value in \( t = Q408 - Q309 \) (a partial recovery), which it is.
evaluated at the extracted shocks. The left lower panel applies to PTA countries and shows that the model follows the untargeted margins closely (represented by the dashed lines). This is true for the intensive and extensive margin. The right lower panel shows that the model also tracks the non-PTA data well, particularly in the first crisis period.

4.2. Quantification of uncertainty shocks

We now determine what type of uncertainty shocks are consistent with the data; whether they are lower for PTAs; and their quantitative implications for export and variety dynamics.

To identify the determinants of $\hat{U}_t$ extracted from the data we use its definition

$$\hat{U}_t = \left[ \frac{1 + \beta_1 \alpha_0 (\alpha_0) - 1}{\beta_0 \alpha_0 (\alpha_0) - 1} \right]^{1/\gamma_1}. \tag{20}$$

where $\beta_2 \equiv \frac{\beta \gamma}{\beta_0 (\alpha_0)}$. Recall that if we have a fixed distribution $\mathcal{M}$ and parameters $\beta_2$, then $\hat{U}_t$ is fully explained by changes in $\alpha$; in this case, worse export conditions reduce tail risk and would increase $U$. However, in the previous section we find $U$ fell in the first period for non-PTA even as $\alpha$ fell. Thus, there must be an offsetting shock to other determinants of $U$. We examine if there are changes in the distribution parameters of $\mathcal{M}$ that are consistent with the extracted shocks.

This exercise requires a value for $\gamma$, and pre-GTC values for $\alpha_0$ and parameters for distribution $\mathcal{M}_0$. Given these we can solve for changes in the distribution parameters of $\mathcal{M}(\alpha_0, H_0(\alpha))$ that are consistent with the observed $\{\hat{a}_t, \hat{U}_t\}$. We can then use these uncertainty shocks to compute counterfactual paths for trade margins at different uncertainty parameters.

4.2.1. Initial uncertainty parameters

In order to link $\hat{U}_t$ to changes in uncertainty parameters we parameterize the distribution. Since (20) is non-linear we require pre-GTC values for $\{m_0, H_0, \gamma_0\}$. We constrain the underlying initial parameters to be common across countries, which reduces the average absolute discrepancy for PTA between the model computed and the data computed margins is 1 percentage point for both the intensive margin and the extensive margin.
free parameters and is consistent with a null-hypothesis of no policy risk before the crisis (or identical risk across countries). To further minimize the number of parameters, we set \( m_0 = 1 \) and assume \( H_0 \) is lognormal (\( \mu_0 = -\frac{\Sigma_0}{2}, \Sigma_0 \)), so we implicitly normalize \( a_0 \) to have a mean of one.

We obtain the initial parameter values as follows. We assume \( \Sigma_0 = 1/8 \); this is also the standard deviation of annual GDP growth over the full period in PTA countries (it is only slightly higher for non-PTA), which is consistent with the model’s main source of variation in \( a \) in the absence of policy risk. We then use the pre-GTC data to calibrate \( \gamma_0 \) and \( a_0^g \) for \( g = \text{PTA}, \text{non-PTA} \). Using the same procedure as before, we extract shocks, \( \delta_{\text{pre}}^g = \frac{a_{\text{pre}}^g}{a_0^g} \), e.g. from Q307 to Q308, and \( \delta_{\text{pre}}^g = \frac{U_{\text{pre}}^g}{U_0^g} \).\(^{24}\)

The procedure yields a set of solutions where \( \gamma \in [0.93, 0.99] \), and we choose the median value of these so \( \{\gamma_0, a_{\text{pre}}^\text{PTA}, a_{\text{pre}}^\text{non-PTA}\} = \{0.94, 1.20, 1.38\} \).

### 4.2.2. Uncertainty parameter shocks

Combining (20) with the initial parameters we extract shocks to uncertainty parameters. We keep \( \gamma \) constant and focus on changes in the mixing weights. If exporters place a higher weight \( m_a \) on a distribution with \( \Sigma > \Sigma \)—a MPS of the initial one, then export risk increases. To recover the underlying uncertainty shocks we use (20). We take the log average of the RHS of (20) over the four quarters of the period evaluated at the extracted shocks to \( a \) and equate it to the average of \( \ln U_t \) extracted from the data.

We first apply the procedure for non-PTAs in the first crisis period. Since we have two parameters to calibrate with a single expression, we set \( \Delta m_2^\text{non-PTA} = 1 \) and find that a value of \( \Delta \Sigma = 0.68 \) can explain \( U_t \). To determine if there was a smaller increase in risk for the PTA in the same period, we apply the same method, but now we fix \( \Sigma = 0.68 \) and solve for the change in weight. We find \( \Delta m_2^\text{PTA} = 0.33 \), so the increase in export risk to PTAs was 1/3 smaller than for non-PTAs in the first crisis year.

What can account for the large differential increase in risk for non-PTA? In a model without policy uncertainty, this must reflect an economic risk 5 times higher than the initial \( \Sigma \) we use, 0.68/(1/8). An increase of this magnitude for economic risk seems unreasonably large and suggests there was an increase in other risks. Can the increase in export risk be fully accounted by a reasonable change in the trade policy distribution? Using the relationships in Section 3.3.2 and \( \tau = \tau^{-1} \), the required change in the mean of \( \ln \tau \) is \( \Delta \mu = -\Delta \mu/\alpha = 0.075 \).\(^{25}\) Assuming no correlation of shocks then \( \Delta \Sigma^2 = \Delta \Sigma^2/\alpha^2 = 0.05 \), or less if tariffs and income shocks are negatively correlated. These are significant parameter changes, but also reasonable if U.S. exporters believed the tariff distribution in non-PTA markets was moving to reflect a trade war.\(^{26}\) The finding that the probability of moving to this riskier distribution was only 1/3 for PTA markets is consistent with these agreements having a lower probability of trade wars.

In the second and third years of the crisis period the uncertainty factors no longer have large unexplained components when evaluated at the original \( \Sigma \) and thus do not require increased weights on riskier distributions. This is consistent with the subsiding fear of a trade war after 2009.

### 4.2.3. Uncertainty counterfactuals

Using the initial values for \( \{m_0, \gamma_0, a_0\} \) we can compute counterfactuals that quantify the role of uncertainty shocks. We consider two counterfactual dynamic paths: (i) no shock to the initial uncertainty parameters and (ii) persistently higher uncertainty parameters equal to those recovered in the first year of the crisis. We focus on the extensive margin where most of the PTA differentials are present.

The counterfactual path with constant initial uncertainty parameters reflects only changes in export conditions, \( a \). We obtain it by computing \( \bar{U}_t = \bar{U}_{\tau = \tau_0, \bar{a}_t} \) in Eq. (20) and then recalculating cutoffs and the implied sequence of midpoint growth rates.

The top right panel of Fig. 10 plots this counterfactual for the extensive margin (pointed purple line). To evaluate the impact of removing these uncertainty shocks, we also plot the path implied by the model that includes those shocks (solid purple line). We see that uncertainty shocks lowered export growth by 3.4 pp for the extensive margin for non-PTAs in the first year on average, and as much as 6 pp in the second quarter of 2009. The left upper panel shows a similar pattern for the PTA, but uncertainty shocks have a smaller impact: they reduce export growth by 1.2 pp in the first year. In the second year, the uncertainty shocks increased export growth, indicating there was an uncertainty reversal, which was larger for non-PTAs. Uncertainty shocks have no significant impact in the final year. This shows the moderating effect of PTAs on uncertainty shocks on exports via the extensive margin. There is no significant effect via the intensive margin for PTA or non-PTA.

What if the higher uncertainty in the first period had persisted? To answer this, we use the uncertainty parameters \( r_t \) for the first crisis year instead of \( r_0 \) and then follow the same procedure as the counterfactual above applied to the second and third year of the crisis. In the lower right panel of Fig. 10 we plot this counterfactual for non-PTA. We also plot the model path at the observed uncertainty parameters and find that it is higher for the last two years by 4.3 pp on average and persists even in 2011Q3. That differential was only about 2.2 pp for the PTA, reflecting the lower probability we estimated of riskier export conditions.

\(^{24}\) Specifically we have \( \bar{U}_{\text{pre}} = \left[1 + \frac{\ln \left(\frac{m_0(a_{\text{pre}}^g)}{a_0^g}\right) - 1}{1 + \ln \left(\frac{m_0(a_{\text{pre}}^g)}{a_0^g}\right) - 1}\right] \), and take the log average over four quarters of the shocks on the LHS and of the model expression on the RHS evaluated using the shocks \( a_{\text{pre}} \) to solve for the set of \( \gamma \) and \( a^g \) that satisfy the equality. Since we impose \( \gamma^\text{PTA} = \gamma^\text{non-PTA} \) the calibrated values also incorporate this restriction.

\(^{25}\) Since we focus on a MPS of \( a \) we have \( \Delta \mu = -\Delta \Sigma^2/2 \).

\(^{26}\) The implied growth in the mean of \( \tau \) is \( \ln E(\tau)/E(\tau) = 0.125 \) (if no correlation), which is not far from the realized changes in Chinese tariffs on the U.S. in the recent trade war.
interpretation is that reversing the probability of a trade war in the last two years increased exports to non-PTA by 4.3 pp and 2.2 pp for PTAs.

4.2.4. Uncertainty contribution and unexplained residuals

We compare the model-implied contribution of uncertainty shocks to the unexplained extensive margin from Section 2. In Fig. 11, we plot the same unexplained residual as in Fig. 6 (dotted line). This residual reflects any determinants of the extensive margin orthogonal to the destination GDP and its deflator, so it can include shocks to uncertainty parameters. The contribution of uncertainty parameter shocks is based on the counterfactual previously discussed and illustrated at the top of Figure 10. Specifically, we define the uncertainty contribution as the difference between the model estimate with and without a change in the mixing weights; it is represented by the purple line.

The model’s uncertainty contribution to the extensive margin tracks the path of the unexplained residuals closely, particularly in the first two years of the GTC where the correlation of the series is over 0.9 for either PTA or non-PTA. The average residual and uncertainty contributions have the same sign for any given crisis period. Moreover, in the first two periods the magnitude of the uncertainty contribution is not negligible relative to the residual.27 For example, in the first year the uncertainty shocks contributed -3.4 pp to non-PTA growth, whereas the unexplained residual was -4.6 pp.28

4.2.5. Robustness

We examine the robustness of the model fit to three alternative parameterizations. First, we increase the role of the death rate $\delta$ from 0.09 to 0.13 and decrease that of the capital depreciation shock $d$ to maintain a constant exit rate $\beta = 0.76$. Second, we increase the elasticity of substitution $\sigma$ from 3 to 4. Third, we increase the Pareto parameter $k$ from 4 to 4.5. In each case, we recovered the shocks and computed the new margins as described in Section 4.1.

The fit of the model relative to the data is very similar to the benchmark. In the benchmark the correlation between the data and respective model margins was above 0.98 for PTA and 0.95 for non-PTA. These correlations are the same for each of the robustness scenarios. Furthermore, the calibrated margins respond to the changes in the parameters as expected. For example, increasing the role of the death rate $\delta$ strengthens the extensive margin responses and weakens those of the intensive margin.29

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27 The mean absolute value of the residual in the first two years is at least twice as high as its mean absolute deviation from the uncertainty shock.

28 In the last year the uncertainty shocks do not contribute to growth significantly and thus do not explain the residual, which is common across destinations and thus likely to reflect a U.S. global shock not present in the model.

29 Additionally, taking the original calibrated values of $\gamma, \Sigma, \text{and } m'$ we explore the robustness of the uncertainty contribution for the extensive margin for the different values of $(d, \delta, k, \sigma)$. We followed the counterfactual approach $\tilde{U}_t (r_t = \tilde{r}_t, \tilde{a}_t)$ in Section 4.2.4 and we find that our results are robust to the different scenarios outlined above.
5. Conclusion

We examine the impact of demand uncertainty on firm export dynamics and the role of trade agreements in mitigating them. We provide a novel set of stylized facts for U.S. export dynamics that contributes to understanding the GTC and subsequent recovery. We also identify a larger extensive margin response in destinations with no PTA with the U.S. The theory extends the research on trade policy uncertainty. It examines interacting risks and the dynamics of exporting more broadly. We also employ a novel and tractable way to evaluate and quantify the importance of the uncertainty mechanism, which can be applied in other settings.

The model tracks the untargeted dynamics of the extensive and intensive margins well and uncovers significant uncertainty shocks during the GTC. These shocks include a large increase in the export risk parameter in the first year of the crisis for non-PTA destinations, which was three times larger than for PTAs, and then reversed after 2009Q4. We argue that this is consistent with a change in policy parameters and the timing of trade war threats: high at the start of the crisis and then receding. The implied uncertainty shock contribution to the extensive margin accounts for a large fraction of the unexplained differential non-PTA decline in the first year of the crisis.

These findings highlight the insurance value of PTAs during economic crises, a benefit of the rules-based trading system more broadly that is often emphasized by businesses and policymakers. The insurance value of PTAs contributes to their large trade effects and should be taken into account when deciding to enter or exit such agreements.\[^{30}\]

Declaration of Competing Interest

None

Appendix A. Data appendix

**Firm-level Trade:** Our primary source is the Longitudinal Foreign Trade Transactions Database (LFTTD). This links U.S. import and export transactions to the firms in the Longitudinal Business Database (LBD), which covers the universe of non-farm private sector employers in the U.S. We construct measures of the number of firms exporting to a particular country and at the product level by month.

We measure entry, exit, and export growth for each quarter relative to the same quarter in the previous year. We use the following definitions to construct the decomposition in Eq. (2).

- **Firm:** A firm is a single or multi-unit enterprise as defined in the Business Register (Standard Statistical Establishment List). Trade flows not matched to a firm are dropped.
- **Firm-Product Variety:** We concord 10 digit Schedule B export commodity codes (6 digit Harmonized System + 4 digit statistical classification) using the method of Pierce and Schott (2009). This ensures that entry, exit, and churning of varieties is not the result of spurious re-classification of commodities across statistical codes. We then define varieties within each destination and industry by the firm-product pair.
- **Entry:** A firm or firm-product variety that is traded at time $t$, but was non-traded at time $t - 4$.
- **Exit:** A firm or firm-product variety that is non-traded at time $t$, but was traded at time $t - 4$.

\[^{30}\] Future research should examine additional agreements and mechanisms such as whether PTAs deepen input-output linkages and reduce the risk of protectionism further, as in Blanchard et al. (2016).
• Continuers: A firm or firm-product variety that is traded at both time \( t \) and \( t - 4 \).

**Country Sample and PTA Definition:** By the final year of our analysis in 2011, the U.S. had PTAs with 17 countries, but we exclude a number of them because they were implemented after 2006 in the midst of the recession and GTC. We focus on seven countries that had a PTA in place by 2006 and that had quarterly GDP data available from 2001 to 2011: Israel (1985), Canada (1989), Mexico (1994), Chile (2004), Australia (2005), Guatemala (2006), and Morocco (2006). The set includes developed and developing countries and represents more than 40% of all U.S. exports.

**GDP and Deflator Measures:** We use quarterly GDP data from the IMF International Financial Statistics. All nominal GDP data are converted into U.S. dollars, which therefore incorporates exchange rate variation in demand. We use year-on-year quarterly GDP and deflator growth rates as control variables in the regression Eq. (1) estimated for Fig. 2. These data restrict our sample to the 67 countries that cover most of the U.S. export value in the LFTTD. We weight these measures up by the average pre-crisis country exports of the PTA and non-PTA country groups to compute the unexplained residuals plotted in Fig. 6.

In sum, our data sample accounts for 88% of all transactions by value that are matched to a firm in the LFTTD in an average quarter. So the sample selection due to missing GDP data, PTA switching in the crisis is quantitatively small. The full list of PTA and non-PTA export destinations is in Table A1.31

<table>
<thead>
<tr>
<th>Table A1</th>
<th>List of non-PTA and PTA countries in regression sample</th>
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<tbody>
<tr>
<td><strong>Non-PTA Countries</strong></td>
<td><strong>PTA Countries</strong></td>
</tr>
<tr>
<td>Argentina</td>
<td>Kazakhstan*</td>
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<td>Armenia, Republic of</td>
<td>Korea, Republic of</td>
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<td>Kyrgyz Republic</td>
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<td>Azerbaijan, Republic of</td>
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</tbody>
</table>

Notes: * Not WTO/GATT member during sample period. † Joins WTO in 2008. China is omitted because quarterly GDP is not available from IMF at time of writing.

**Appendix B. Theory appendix**

**B.1. Value functions under uncertainty**

The expected value of starting to export at time \( t \) conditional on observing current conditions \( a_t \) is

\[
\Pi_e(a_t, c, r) = \pi(a_t, c) + \beta(1 - \gamma)\Pi_e(a_t, c, r) + \gamma E\Pi_e(a_t', c, r),
\]

\[ (21) \]

31 The sample corresponds to 70–75% of U.S. total exports since not all trade transactions can be matched to a firm in the LFTTD or the exporter is not part of the non-farm employer universe, e.g. government entities, self-employed, agriculture, etc.
which includes current operating profits upon entering and the discounted future value. Without a shock the firm value next period remains \(I_{t+1}(a_t, c_t, r_t)\). If a shock arrives, then a new \(\alpha'\) is drawn, so the third term is the ex-ante expected value of exporting following a shock, \(\mathbb{E} I_{t+1}(\alpha', r_t) = \mathbb{E}(a', c_t)/(1 - \beta)\), where \(\mathbb{E}\) denotes the expectation over a fixed and known distribution, \(M\). \(\gamma\)

The expected value of waiting is

\[
\Pi_{w}(c, r) = 0 + \beta (1 - \gamma + \gamma M(\bar{a})) \Pi_{w}(c, r) + \beta \mathbb{E}(a, c) + \beta \gamma (1 - \gamma M(\bar{a})) (\mathbb{E} I_{t+1}(a' \geq \bar{a}, c, r) - K).
\]

(22)

A non-exporter at \(t\) receives zero profits from that activity today. The continuation value remains at \(\Pi_{w}\) if either demand is unchanged, with probability \(1 - \gamma\), or changes to some level that is not sufficiently high to induce entry, with probability \(\gamma M(\bar{a})\). If demand changes and is above some endogenous trigger level, \(a' \geq \bar{a}\), then we obtain the third term, reflecting the expected value of exporting net of the sunk cost, \(K\), conditional on the new demand being high enough to trigger entry. The conditional expected value of exporting if \(a' \geq \bar{a}\) is given by

\[
\mathbb{E} I_{t+1}(a' \geq \bar{a}, c, r) = \mathbb{E}(a' \geq \bar{a}, c, r) + \beta (1 - \gamma) \mathbb{E} I_{t+1}(a' \geq \bar{a}, c, r) + \beta \gamma \mathbb{E} I_{t+1}(a', c, r).
\]

(23)

A firm with costs \(c_t\) is indifferent between entering or waiting if demand is at a threshold level \(a_t\), otherwise, in solving for \(a_t = \bar{a}(c_t)\) we characterize the marginal exporting firm at any current demand, which is characterized by a cost parameter \(c_t^{\bar{a}}\) defined by \(a_t = \bar{a}(c_t^{\bar{a}})\). If a firm has costs equal to this threshold, then in that period all other firms in that industry with lower costs also export to that particular destination.

We obtain an expression for this cutoff by using the entry condition in (6); the value functions in (21), (22) and (23), and the expression for \(\mathbb{E} I_{t+1}\). As an intermediate step to gain some intuition we note that for the marginal entrant the sunk cost must equal the following:

\[
K = \frac{\mathbb{E}(a' \geq \bar{a}, c, r)}{1 - \beta (1 - \gamma)} + \frac{\beta \gamma}{1 - \beta} \mathbb{E}(a' \geq \bar{a}, c, r) + \frac{\beta \gamma (1 - \gamma M(\bar{a}))}{1 - \beta} \frac{\mathbb{E}(a' \geq \bar{a}, c, r)}{1 - \beta (1 - \gamma)}.
\]

(24)

If \(\gamma = 0\), then there is no demand uncertainty and \(K = \frac{\mathbb{E}(a' \geq \bar{a}, c, r)}{1 - \beta (1 - \gamma)}\), i.e. it would be equal to the present discounted value of profits evaluated at the current demand, where \(c_t^{\bar{a}}\) is the marginal cost for the marginal entrant when \(\gamma = 0\). If demand can change, then the current profit is discounted at a higher rate that captures the probability of a demand shock; \(K\) must now cover the value of profits until demand changes (first term), plus the expected profits following the change (second term), and the third term, which is the expected loss of entering today given that conditions can eventually improve. This last term is negative, and captures the option value of waiting.

B.2. Cutoff: Single uncertainty state

To derive the cutoff in eq. (7) from the text we rearrange (24) as follows:

\[
\frac{\mathbb{E}(a' \geq \bar{a}, c, r)}{1 - \beta (1 - \gamma)} = \frac{\mathbb{E}(a' \geq \bar{a}, c, r)}{1 - \beta (1 - \gamma)} + \frac{\beta \gamma}{1 - \beta} \mathbb{E}(a' \geq \bar{a}, c, r) + \frac{\beta \gamma (1 - \gamma M(\bar{a}))}{1 - \beta} \frac{\mathbb{E}(a' \geq \bar{a}, c, r)}{1 - \beta (1 - \gamma)}.
\]

(24)

where the second line uses the equilibrium cutoff under no uncertainty, defined by \(K = \frac{\mathbb{E}(a' \geq \bar{a}, c, r)}{1 - \beta (1 - \gamma)}\) and the definition of the profit function. The third re-arranges and the fourth uses the definition of \(\omega\) in (8) (after recognizing that \(\mathbb{E}(a') - (1 - M(a_t)) \mathbb{E}(a' \geq a_t) = M(a_t) \mathbb{E}(a' \leq a_t)\)).

---

\(32\) This term is time invariant because the distribution of future conditions after a shock, \(M(a_t)\), is time invariant so even if there is a new \(\alpha\) at \(t + 1\) this provides no additional information at time \(t\) about future conditions. The conditional mean of \(\alpha\) and the expected value of exporting, \(\Pi_{t+1}(a_t, c_t, r_t)\), vary over time since they depend on current conditions.

\(33\) We can do so since \(\alpha\) is common to all firms exporting to a given market in a given industry and the marginal cost is the only source of heterogeneity among such firms. Assuming a continuum of firms in any given industry with productivity that can be ranked according to a strictly increasing CDF, we can find the marginal export entrant for any \(a_t\).
B.2.1. Proof of Proposition 1

In order to prove Proposition 1, we start by proving a lemma. First, assume that there is only one state, $S = 1$ such that $M(a) = H_s(a)$ and we abuse notation by denoting as $H(a)$. Under this assumption, we state the following definition and lemma:

**Definition: Uncertainty Ranking** $r' = \{y', H'(a')\}$ is more uncertain than $r$ if it has either higher volatility, defined as $\gamma' > \gamma$, and/or risk, defined as $H$ second-order stochastically dominating (SSD) $H'$. 

**Lemma 1: Uncertainty Shocks and Entry** An increase in the demand regime uncertainty reduces net entry: $c_t^U(r') \leq c_t^U(r)$. Moreover, the volatility and risk components of uncertainty have a complementary effect on entry: $\frac{\partial c_t^U(\gamma, H)}{\partial \gamma} \leq \frac{\partial c_t^U(\gamma, H)}{\partial \gamma}$.

We split the lemma into each of the components of the demand regime: $r = \{\gamma, H\}$ as follows.

(a) For given $H$, a riskier demand regime $(\gamma' > \gamma)$ reduces entry: $c_t^U(\gamma') \leq c_t^U(\gamma)$.

Using (7), $S = 1$, and the definition of $U$ we obtain:

$$\frac{\partial \ln c_t^U}{\partial \gamma} = \frac{1}{\sigma - 1} \frac{\partial}{\partial \gamma} \ln \left(1 + \frac{\beta'(\omega(a) - 1)}{1 - \beta'(1 - \gamma)}\right)$$

$$= \frac{1}{\sigma - 1} \frac{\beta(1 - \beta)}{1 - \beta(1 - \gamma)} \frac{\omega(a) - 1}{1 - \gamma \omega(a)} \leq 0$$

Recall that $\beta \in (0, 1)$ and $\omega \geq 0$ so the inequality follows iff $\omega(a) = 1 = \omega(\gamma) = 1 = -H(a) = \frac{a - \omega(a)}{a} \leq 0$, which is true since the CDF $H(a) \leq 1$ and $E' \leq a$ (by definition). Moreover, $c_t^U(\gamma') < c_t^U(\gamma)$ for all $a > a_{\text{min}}$ since then $\omega(a) < 1$.

(b) For given $\gamma > 0$, a riskier demand regime $(H SSD H')$ reduces entry: $c_t^U(H') \leq c_t^U(H)$.

From (7), (8) and (9) we see that $H$ affects entry only through $\omega$, and the latter only affects entry if $\gamma > 0$. Thus, there is (weakly) less entry under $r'$ than an alternative regime $r$ with the same $\gamma$ but a $H$ that SSD $H'$ iff $\omega > \omega'$. To see that is the case we first rewrite $\omega$ as

$$\omega(\gamma) = 1 - H(a) + \frac{H(a)}{a} \int_0^a ah(a) da$$

$$= 1 - H(a) + \frac{1}{a} \int_0^a H(a) da$$

$$= 1 - \frac{1}{a} \int_0^a H(a) da$$

where the first line uses definition of $\omega$, and of the conditional mean and the second uses $h(a) = h(a)/H(a)$ and $dh(a) = h(a) da$. The third line uses integration by parts and the fourth simplifies. We can do the same for $\omega'$, and subtract from $\omega$ to obtain

$$\omega - \omega' = \frac{1}{a} \left[ \int_0^a H'(a) da - \int_0^a H(a) da \right] \leq 0$$

If $H SSD H'$, then the inequality in brackets follows for all $a$, with strict inequality for at least some $a$. The weak inequality in $c_t^U(H') \leq c_t^U(H)$ allows for the possibility that the distributions overlap at low $a$, or if $a = a_{\text{min}}$ and $H$ is a mean preserving compression of $H'$.

The proof of Proposition 1 is simple given what is established in Lemma 1, which corresponds to the special case where $m_a = 1$. First, Lemma 1(a) shows that the cutoff is decreasing in $\gamma$ as in Proposition 1(b). Second, Lemma 1(b) shows that if the distribution of $a$ is $M(a)$, then $\omega(a) - 1 = -\frac{1}{\beta'} \int_0^a M(a) da$ and using the mixture definition of $M$ we obtain $\omega(a) - 1 = -\sum_{s} m_s \frac{1}{\beta'} H_s(a) da$. This is equivalent to $\omega(a) = \sum_{s} m_s \omega_s(a)$ since $\omega(a) - 1 = -\frac{1}{\beta'} \int_0^a H_s(a) da$ and $\sum m_s = 1$. From here, the remaining results from Proposition 1 follow from Lemma 1.

B.2.2. Export elasticity effects in Eq. (11)

Using (10) we obtain

$$\frac{d \ln R(a_t, c_t^U(a_t))}{d \ln a_t} = 1 + \frac{\partial \ln R(a_t, c_t^U)}{\partial c_t^U} \frac{\partial \ln c_t^U}{\partial a_t}.$$

24
Using the definitions of $c^j$ and $U_j$ in Proposition 1 we have

$$\frac{\partial \ln c^j}{\partial \ln a_i} = \frac{\partial \ln c^0}{\partial \ln a_i} + \frac{\partial \ln U_j}{\partial \ln a_i}$$

$$= \frac{1}{\sigma - 1} + \frac{\partial \ln U_j(a_i, r)}{\partial \omega} \frac{\partial \omega}{\partial a_i}$$

$$= \frac{1}{\sigma - 1} \left( 1 + \frac{\gamma^j}{\beta^j(1 - \gamma^j \omega)} \frac{\partial \omega}{\partial a_i} \right)$$

From the definition of $\omega$ in Proposition 1 we obtain $a_i \frac{\partial \omega}{\partial a_i} = -((\omega - 1) + H(a_i)) = -\frac{1}{\omega} \int_0^\omega adH(a) < 0$ so $\frac{\partial \ln U_j}{\partial \ln a_i} < 0$. Replacing $\frac{\partial \omega}{\partial a_i}$ and simplifying we can verify the caution effect is dominated by the deterministic impact:

$$\frac{\partial \ln c^j}{\partial \ln a_i} = \frac{1}{\sigma - 1} \left( 1 + \frac{\gamma^j}{\beta^j(1 - \gamma^j \omega)} a_i \left( -\frac{1}{a_i}((\omega - 1) + H(a_i)) \right) \right)$$

$$= \frac{1}{\sigma - 1} \left( 1 - \frac{\gamma^j(\omega - 1 + H(a_i))}{\beta^j(1 - \gamma^j \omega)} \right) > 0$$

where the inequality holds since the term in is positive for all $\beta, \gamma$ in $[0, 1]$ given $H(a_i) \leq 1.$

B.3. Proof proposition 2

Export risk averse objective and risk reducing PTA

By definition, the exporter government has a PTA motive if there is some change in $\{\Delta^\text{PTA}, \Delta^\text{SSD}\}$ s.t. $G^\text{PTA} > G^\text{M}$. We define a government as export risk averse if $G(a_i, M) \gamma \geq G(a_i, M'(a_i, \gamma)$ for all $a_i$ whenever $M$ SSD $M'$ (with equality at $\gamma = 0$). In Lemma 1 we show $M$ SSD $M'$ is equivalent to $\omega_i(a_i) \geq \omega_i(a_i)$ for all $a_i$ a similar condition holds for the mixture case used in Proposition 1. So the risk averse government benefits from a $\Delta^\text{PTA} = m^\text{PTA} - m$ such that

$$m^\text{PTA} \omega_i(a_i) + (1 - m^\text{PTA}) \omega_i(a_i) \geq m \omega_i(a_i) + (1 - m) \omega_i(a_i)$$

(26)

$$\omega_i(a_i) - \omega_i(a_i) \Delta^\text{PTA} < 0.$$

(a) PTA and stationary export values at given $a_i$

We show the PTA increases cutoffs and this implies higher stationary export values and firms. We use the cutoff in (7), which is increasing in $\omega$ (Proposition 1b). This cutoff is higher under a PTA characterized by an insurance effect since, as shown above, it is characterized by $\omega^\text{PTA} > \omega$ if $\gamma > 0$. The partial effect is given by

$$\frac{\partial \ln c^j}{\partial m} \Delta^\text{PTA} = \frac{\partial \ln c^j}{\partial \omega} \frac{\partial \omega}{\partial m} \Delta^\text{PTA} = \frac{\beta^j \gamma}{1 - \beta^j(1 - \gamma^j \omega)} \frac{\omega_i(a_i) - \omega_i(a_i)}{\sigma - 1} > 0,$$

(27)

where the inequality is due to (26), $\omega \in (0, 1)$ and $\sigma > 1$.

The stationary value for the number of exporters is $nF(c^j(a_i, M))$ where $n$ is the exogenous mass of domestic firms and $F$ is the cost CDF so it is increasing in $c^j$.

The stationary value for export value is $R(a_i, M)$ given in (10), also increasing in the cutoff, via the upper limit.

(b) PTA and expected log stationary values

The difference in the expected log number of stationary exporting firms is given by the LHS of Eq. (13) since $n$ is exogenous; it reflects the difference in $\Delta^\text{PTA}$ due to the PTA that affects $M$ and thus the cutoff as shown in part (a) over which we compute the average using $M$ for the PTA and $M'$ otherwise. To obtain the decomposition on the RHS of (13) we simply add and subtract the term $\int_0^\infty F(c^j(a_i, M)) dM$, the cutoff under a PTA when averaged using the non-PTA distribution. The cutoff effect in (13) averages the cutoff differences, which are positive as shown in (a). A similar expression and argument applies to the export values, which are decomposed as
\[ \mathbb{E}_M \ln R\left(a, c^{ij}(a, M')\right) - \mathbb{E}_M \ln R\left(a, c^{ij}(a, M)\right) = \int_a \frac{R\left(a, c^{ij}(a, M)\right)}{R\left(a, c^{ij}(a, M')\right)} dM' + \int_a \frac{R\left(a, c^{ij}(a, M)\right)}{d(M - M')} dM' \quad (28) \]

The compression effects compare averages of a function of \( a \) over two distributions and we recall \( M \) SSD \( M' \). We can thus use the general result

\[ \int_a X(a)\, d(M(a) - M'(a)) \geq 0 \text{ if } d^2X/da^2 \leq 0. \]

So a sufficient condition for the compression effect to be non-negative is for \( X \) to be concave over the range of \( a \).

- Exporters. Using \( X(a) = \ln F(c^{ij}(a)) \) we see log-concavity of \( F \) in \( a \) is equivalent to \( d^2X/da^2 \leq 0 \). We now show that \( \frac{d^2 X}{da^2} \leq 0 \) is sufficient for \( d^2X/da^2 \leq 0 \).

\[ \frac{d^2 X}{da^2} \leq 0 \text{ if } \frac{d^2 X}{da^2} \leq 0 \]

where in the last line the term is non-positive iff \( \frac{d^2 X}{da^2} \leq 0 \) and the first term is negative since \( k(c) > 0 \) (continuous increasing distribution) and \( \frac{d^2 X}{da^2} < 0 \) (shown in log-concavity lemma below).

- Values. Using \( X(a) = \ln R \) defined in (10) we show that \( dk(c)/dc \leq 0 \) is sufficient for \( d^2X/da^2 \leq 0 \) (we omit \( M \) since it applies to any distribution of \( a \))

\[ \frac{d^2 X}{da^2} \leq 0 \text{ if } \frac{d^2 X}{da^2} \leq 0 \]

where \( k(c) = \frac{\partial R(a, c^{ij}(a))}{\partial c^{ij}(a)} = \frac{\partial R(a, c^{ij}(a))}{\partial c^{ij}(a)} > 0 \). Note that in the last line the first term is negative and the second is non-positive as we show in the log-concavity of \( c^{ij} \) lemma below. Therefore, it is sufficient (but not necessary) for the third term to be non-positive, which is ensured by \( dk(c)/dc \leq 0 \).

Lemma (Log-concavity of \( c^{ij}(a) \)): \( \frac{d^2 \ln c^{ij}}{da^2} \leq 0 \)

Without uncertainty this is clear since \( \frac{\partial R(a, c^{ij}(a))}{\partial c^{ij}(a)} |_{c^{ij}=0} = - \frac{1}{\sigma} \left(1 - \frac{1}{\gamma} \right) \). Moreover, it will also hold for any small enough \( \gamma \). We now show it is true for any \( \gamma \) and given by:

\[ \frac{d^2 \ln c^{ij}}{da^2} = - \frac{1}{a^2} \left( \frac{ah(\gamma)}{1 - \beta(1 - \gamma(1 - H))} + \left(1 - \beta(1 - \gamma(1 - H)) \right) \right) < 0 \quad (29) \]
This reduces to the negative expression without uncertainty— the term before — if we set $\gamma = 0$. The term in is positive and thus the inequality follows since $1 > \beta(1 - \gamma \omega)$. We derive it as follows.

$$
\frac{\partial}{\partial a} \frac{d}{d a} \ln c^U = \frac{\partial}{\partial a} \left( \frac{\partial \ln U}{\partial a} + \frac{1}{\sigma - 1} \frac{\partial \ln a}{\partial a} \right)
$$

$$= \frac{1}{\sigma - 1} \left( \frac{\partial}{\partial a} \left( \frac{\beta \gamma}{1 - \beta(1 - \gamma \omega)} \frac{\partial \omega}{\partial a} + a^{-1} \right) \right)
$$

$$= \frac{1}{\sigma - 1} \frac{\partial}{\partial a} \left( \frac{(1 - \beta(1 - \gamma(1 - H)))}{1 - \beta(1 - \gamma \omega)} a^{-1} \right)
$$

$$= \frac{1}{\sigma - 1} \left( \left( -\beta \frac{\gamma h}{1 - \beta(1 - \gamma \omega)} \right) \frac{(1 - \beta(1 - \gamma(1 - H)))}{(1 - \beta(1 - \gamma \omega))^2} \frac{\partial \omega}{\partial a} \right) + \left( \frac{(1 - \beta(1 - \gamma(1 - H)))}{1 - \beta(1 - \gamma \omega)} \right) \left( -\frac{1}{a^2} \right)
$$

$$= -\frac{1}{a^2(1 - \beta(1 - \gamma \omega))} \left( \beta \gamma + \frac{\partial}{\partial a} \ln c^U \right)(1 - \beta(1 - \gamma(1 - H)))$$

$$< 0$$

where the third line uses $-\frac{\partial}{\partial a} \ln c = -(\omega - 1 + H(a)) \frac{1}{\beta}$; we omit algebra and re-arranging terms between line 4 and 5, and the last equality uses $-\frac{\partial}{\partial a} \ln c = \left( 1 + \frac{\beta \gamma}{(1 - \beta(1 - \gamma \omega))} a \frac{\partial a}{\partial a} \right) \frac{1}{\sigma - 1}$.

The inequality follows because $\frac{\partial}{\partial a} \ln c = -\frac{\beta}{\sigma - 1}$ as shown before and $1 > \beta(1 - \gamma \omega), 1 > \beta(1 - \gamma(1 - H))$. Using the definitions for $\frac{\partial}{\partial a} \ln c$ and $\frac{\partial a}{\partial a}$ we obtain the expression in (29).

**Lemma:** Distributions satisfying non-increasing entry condition

Non-increasing entry elasticity: $\frac{d}{d a} \frac{d}{d \ln c} f(c) / d \ln c \leq 0$. This can be directly verified for any differentiable $F$. It holds with equality over the range only for power functions $(c/c_{\text{max}})^k$. Distributions where the inequality is strict include Frechet ($\exp(-c^{-k})$), Weibull ($1 - \exp(-c^k)$), exponential ($1 - \exp(-\lambda c)$), log-normal ($\Phi\left( \frac{\ln c - \mu}{\sigma} \right)$). Non-increasing export entry elasticity: $\frac{d}{dc} \frac{d}{d \ln c} f(c) = \frac{d}{dc} k(c) \leq 0$. Evaluating and simplifying the derivative we can show this condition is equivalent to requiring the density not to rise too fast in the following sense:

$$
\frac{d}{dc} \frac{d}{d \ln c} f(c) \leq 0 \Leftrightarrow \frac{d}{d c} f(c) / d \ln c^{\sigma - 1} \leq k(c)^{\sigma - 2}.
$$

(30)

For example, in the empirically relevant range for $\sigma \geq 2$ the condition is immediately satisfied for any non-increasing density, $\frac{d}{dc} f(c) \leq 0$. It is also satisfied for other distributions. For example, under Pareto the elasticity is constant, $k(c) = k - (\sigma - 1)$, and we can also verify that our condition yields an equality since the LHS $\frac{d}{d \ln c} f(c) = k - 1$. For other distributions we must evaluate $k(c) = \frac{(\ln c)^{\sigma - 1} f(c)}{\int_0^\infty c^{\sigma - 1} dF(c)}$ and check the inequality. Limão and Xu (2021) show it holds for the distributions already listed for non-increasing elasticity.
Appendix C. Quantification and calibration appendix

C.1. Parameter values for quantification

C.1 Parameter values for quantification.

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<tr>
<td>Productivity dispersion</td>
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<td>Export discount factor</td>
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C.1.1. Source and consistency of external parameters with other evidence

The elasticity of substitution is the median value from (Broda et al., 2008), rounded. The distribution of export sales without uncertainty follows a Pareto with shape $k/\sigma - 1$. Our baseline parameters imply a value of 2 and the robustness considers alternatives $k/\sigma - 1 = \{1, 3.3, 2.25\}$. These are within the range estimated in the literature and used for quantification (e.g. Melitz and Redding, 2015, use 1.42). It is also consistent with our own estimates using U.S. export data.

The death rate $\delta$ in the model captures the U.S. firm exit rate. The value used is the one obtained by Broda and Weinstein (2010) for the yearly exit of U.S. manufacturers from scanner data (Table 4). It is also similar to the Business dynamic survey estimate for establishment exit in 2007 (0.08).

The export discount factor is consistent with different pieces of evidence. First, Broda and Weinstein (2010) also compute the yearly exit rate for varieties within the U.S., which is 0.24 (Table 3), which is consistent with our own estimates using U.S. export data.

The export discount factor is consistent with different pieces of evidence. First, Broda and Weinstein (2010) also compute the yearly exit rate for varieties within the U.S., which is 0.24 (Table 3), which is consistent with our own estimates using U.S. export data.

C.2. Expressions for targeted moments and extracted shocks

Starting from the expressions for $(R_t, N_t)$ in equation (18) we derive expressions relating the targeted moments $(\tilde{R}_t, \tilde{N}_t)$ to the extracted shocks $(\tilde{a}_t, \tilde{c}_t)$. The $\bar{x}$ variables denote ratios relative to the respective quarter pre-crisis value and $\bar{x}$ denote annual percent growth rates.

In the third period, the economy has fully recovered and there are no legacy firms. Hence, we use the following expressions for the targeted moments where $t = 3$:

$$N_t = \left( (\tilde{c}_1^t + 1)^k - 1 \right)$$

$$\hat{R}_t = - (\tilde{a}_1 + 1) \left[ (1 - (\tilde{c}_1^t + 1)^k)^{-\alpha - 1} - 1 \right]$$

In period 1 or 2 of the GTC we verify the economy is in either an initial decline or a recovery. Thus, we must account for legacy firms and obtain the following expressions for the targeted moments where $t = \{1, 2\}$

$$N_t = \left( 1 - \beta^t \right) \left( (\tilde{c}_1^t + 1)^k - 1 \right)$$

$$\hat{R}_t = - (\tilde{a}_t + 1) \left[ (1 - (\tilde{c}_t^t + 1)^k)^{\alpha - 1} \right] \left( 1 - \beta^t \right) - 1$$

Using $(\tilde{a}_t, \tilde{c}_t)$ we retrieve the uncertainty factor using (7), which has the same expression in all periods

$$\left( \tilde{c}_1^t + 1 \right)^k = \left( \tilde{a}_1 + 1 \right) \left( \tilde{N}_t + 1 \right)^{\alpha - 1} \left( \tilde{R}_t + 1 \right)^{-\alpha}$$

28
Note that we the annual percent growth rates $\dot{x}$ and the ratios relative to pre-crisis level $x$ are related as follows:

- **Period 1**: $\dot{x}_t = \dot{x}_t + 1$
- **Period 2**: $\dot{x}_{t+1} = \left(\dot{x}_{t+1} + 1\right)\dot{x}_t$
- **Period 3**: $\dot{x}_{t+2} = \left(\dot{x}_{t+2} + 1\right)\dot{x}_{t+1}$

### C.3. Expressions for untargeted moments

After recovering the shocks to $a_t$, $c_t$ using the targeted moments $\dot{N}_t$ and $\dot{R}_t$, we use the midpoint growth rate components as our untargeted moments.\(^{34}\) The midpoint formula can then be expressed as follows:

$$\dot{R}_t^{\text{mid}} = s_1^{\epsilon}R_t^{\epsilon\text{CONT}} + 2\left(s_1^{EN} - s_1^{EX}\right)$$  \hspace{1cm} (31)

where $s_1^c = \frac{R_t^{\epsilon} + R_t^{\sigma \epsilon}}{R_t^{\epsilon}}$ are the corresponding weights and $x = \{c, EN, EX\}$. We derive expressions for each of the components in (31) where we incorporate the role for legacy firms and the different histories that the economy might follow.

**Period 1**: The economy is in an initial decline. Hence, we use the following expressions for the untargeted moments:

- **Continuing Margin**:

  $$s_1^{\epsilon}R_t^{\epsilon\text{CONT}} = 2\frac{\dot{a}_t}{R_t + 1}\left(1 - \delta\right)\left(1 - d\int_0^{\dot{c}_0} c_1^{1 - \alpha} dF(c)\right)$$  \hspace{1cm} (32)

  Applying the Pareto assumption and switching to the $\dot{x}$ notation:

  $$s_1^{\epsilon}R_t^{\epsilon\text{CONT}} = 2\frac{\dot{a}_t - 1}{R_t + 1}\left(1 - \delta\right)\left(1 - d\int_0^{\dot{c}_0} c_1^{1 - \alpha} dF(c)\right)$$

  $$= 2\frac{\dot{a}_t - 1}{R_t + 1}\left(1 - \delta\right)\left(1 - d\left(1 - C_t\right)\right)$$

  $$= 2\frac{\dot{a}_t - 1}{R_t + 1}\left(1 - \delta\right)\left(C_t + 1 - d\left(1 - C_t\right)\right)$$

  where the last line adds and subtracts $C_t$ inside the parenthesis to ease the interpretation.

- **Net Entry Margin**:

  $$s_1^{EN} - s_1^{EX} = \frac{\delta\dot{N}_t - (1 - \delta)d\int_0^{\dot{c}_0} c_1^{1 - \alpha} dF(c)/\int_0^{\dot{c}_0} c_1^{1 - \alpha} dF(c)}{R_t + 2}$$  \hspace{1cm} (33)

  where $\dot{N}_t$ is export growth in the stationary state at the current cutoff.

Applying the Pareto assumption and switching to the $\dot{x}$ notation:

$$s_1^{EN} - s_1^{EX} = \frac{\delta\dot{C}_t - \left(\delta + (1 - \delta)d\left(1 - C_t\right)\right)}{R_t + 1}$$  \hspace{1cm} (34)

Collecting all the terms:

$$\dot{R}_t^{\text{mid}} = s_1^{EN}R_t^{\epsilon\text{EN}} + s_1^{EX}R_t^{\epsilon\text{EX}} + s_1^{\epsilon\text{CONT}}R_t^{\epsilon\text{CONT}}$$

$$= \frac{2}{R_t + 1}\left[\delta\dot{a}\dot{C}_t - \left(\delta + (1 - \delta)d\left(1 - C_t\right)\right) + (\dot{a} - 1)(1 - \delta)\left(C_t + (1 - d)(1 - C_t)\right)\right]$$

**Period 2**: The economy is recovering, but not yet fully recovered. Hence, we use the following expressions for the untargeted moments:

\(^{34}\) Detailed derivations can be found in the online appendix at https://doi.org/10.1016/j.jinteco.2022.103661.
• Continuing Margin:

\[ s^2R^\text{CONT}_{t+1} = 2 \frac{a_{t+1}/a_t - 1}{R_{t+1}/R_0 + 1} \frac{a_t/a_0}{R_t/R_0} \left( (1 - (1 - \delta)(1 - d)) \left( \frac{\gamma^{k-(\alpha-1)}}{C_0^k} \right) + (1 - \delta)(1 - d) \left( \frac{\gamma^{k-(\alpha-1)}}{C_0^k} \right) \right) \]  

\[ (1 - d)(1 - d) \left( \frac{\gamma^{k-(\alpha-1)}}{C_0^k} \right) + (1 - d) \right) \]  

(35)

• Net Entry Margin:

\[ 2(s^E_{t+1} - s^X_{t+1}) = \frac{2}{R_{t+1}/R_0 + 1} \left( \frac{a_{t+1}}{a_0} \right) \left( \frac{\gamma^{k-(\alpha-1)}}{C_0^k} \right) \]  

\[-\left( \frac{R_t/R_0}{d(1 - \delta)^2(1 - d)(1 - d) \left( 1 - \frac{a_{t+1}}{a_0} \right) \left( \frac{\gamma^{k-(\alpha-1)}}{C_0^k} \right) \right) \right) \]  

where \( \Theta = (1 - \beta^2 - \beta(1 - \delta)d) \)

**Period 3 - Without Full Recovery**: The economy is recovering, but not yet fully recovered. Hence, we use the following expressions for the untargeted moments:

• Continuing Margin:

\[ s^2R^\text{CONT}_{t+2} = 2 \frac{a_{t+2}/a_t - 1}{R_{t+2}/R_0 + 1} \frac{a_t/a_0}{R_t/R_0} (1 - \delta) \left( 1 - (1 - \beta^2)^2 \left( \frac{C_{t+1}^0}{C_0^0} \right)^k - (\alpha-1) \right) \]  

\[ + (1 - \beta^2)^2 \left( \frac{\gamma^{k-(\alpha-1)}}{C_0^k} \right) \]  

\[-(1 - \frac{a_{t+1}}{a_0}) \left( 1 - \frac{\gamma^{k-(\alpha-1)}}{C_0^k} \right) \right) \]  

(37)

• Net Entry Margin:

\[ 2(s^E_{t+2} - s^X_{t+2}) = \frac{2}{R_{t+2}/R_0 + 1} \left( \frac{a_{t+2}/a_0}{R_{t+2}/R_0 + 1} \right) \left( \delta - \Theta \left( \frac{C_{t+2}^0}{C_0^0} \right)^k - (\alpha-1) \right) \]  

\[ + \Theta \left( \frac{C_{t+2}^0}{C_0^0} \right)^k - (\alpha-1) \]  

\[ - \left( \frac{R_t/R_0}{d(1 - \delta)^2(1 - d)(1 - d) \left( 1 - \frac{a_{t+1}}{a_0} \right) \left( \frac{\gamma^{k-(\alpha-1)}}{C_0^k} \right) \right) \right) \]  

where now \( \Theta = (1 - \beta^3 - \beta^2(1 - \delta)d) \).

**Period 3 - With Full Recovery**: The economy has now fully recovered from the initial decline. Hence, we use the following expressions for the untargeted moments:

• Continuing Margin:

\[ s^2R^\text{CONT}_{t+2} = 2 \frac{a_{t+2}/a_0 - a_{t+1}/a_0}{R_{t+2}/R_0 + R_{t+1}/R_0} \left( 1 - \delta \left( \frac{C_{t+1}^0}{C_0^0} \right)^k - (\alpha-1) \right) \]  

\[ + \beta^2 \left( 1 - \frac{C_{t+1}^0}{C_0^0} \right)^k - (\alpha-1) \]  

(39)

• Net Entry Margin:

\[ 2(s^E_{t+2} - s^X_{t+2}) = \frac{2}{R_{t+2}/R_0 + R_{t+1}/R_0} \left( a_{t+2}/a_0 \right) \left( \delta - \Theta \left( \frac{C_{t+1}^0}{C_0^0} \right)^k - (\alpha-1) \right) \]  

\[ + \Theta \left( \frac{C_{t+1}^0}{C_0^0} \right)^k - (\alpha-1) \]  

\[- \left( \frac{R_t/R_0}{d(1 - \delta)^2(1 - d)(1 - d) \left( 1 - \frac{a_{t+1}}{a_0} \right) \left( \frac{\gamma^{k-(\alpha-1)}}{C_0^k} \right) \right) \right) \]  

where \( \Phi = (1 - \beta^2(1 - \delta)) \)
C.4. Expressions for uncertainty counterfactuals

In the counterfactuals exercises, we start from the expressions for $\tilde{U}_t$ that allows us to explore which parameters are driving the recovered shocks to $U_t$,

$$
\tilde{U}_t = \left[ \frac{1 + \beta_t [\tilde{\omega}_t (a_t a_0, r_t) - 1]}{1 + \beta_t [\tilde{\omega}_t (a_t a_0, r_t) - 1]} \right]^{\frac{1}{\lambda}}.
$$

where $\beta_t = \frac{\beta \gamma}{\lambda (1 + \gamma)}$ and we have made explicit that $\tilde{\omega}_t$ depends on $a_t$ and $r_t$.

In the first counterfactual exercise, we set $\tilde{U}_t^{1} = \tilde{U}_t (a_t a_0, r_t = 0)$. This implies that the uncertainty regime $r_t$ remains unchanged at its pre-GTC value. Note that we allow $\tilde{U}_t^{1}$ to adjust due to changes in $a_t a_0$. Then we feed this $\tilde{U}_t^{1}$ into the midpoint formulas derived above to obtain the counterfactual path for the continuing and net entry margins.

In the second counterfactual exercise, we set $\tilde{U}_t^{2} = \tilde{U}_t (a_t a_0, r_t = 1)$ for $t = (2, 3)$. This implies that the uncertainty regime $r_t$ remains unchanged at its initial GTC value. Similarly, we then use this $\tilde{U}_t^{2}$ to compute the counterfactual path for the continuing and net entry margins.

C.5. Other calibration details

The calibration procedure requires pre-GTC values for the $r_0 = \{m_0, H_0, \gamma_0\}$ for PTA and non-PTA countries. We assume that there are no differences in the $r_0$ for the PTA and non-PTA countries. Furthermore, we assume that $m_0 = 1$ and $H_0 = \ln N(-\Sigma_0^2, \Sigma_0)$ and set $\Sigma_0 = 1/8$. Using the formulas for the targeted moments for period 3, we recover pre-GTC values for $\tilde{a}_\text{pre}, \tilde{c}_\text{pre}, \tilde{U}_\text{pre}$ for the PTA and non-PTA countries.

After recovering $\tilde{U}_\text{pre}$ then we use the data to inform the value of $\gamma_0$ by taking the log average over four quarters of

$$
\tilde{U}_\text{pre} = \left[ \frac{1 + \beta_0 [\hat{\omega}_0 (a_0 a_0, r_0) - 1]}{1 + \beta_0 [\hat{\omega}_0 (a_0 a_0, r_0) - 1]} \right]^{\frac{1}{\lambda}}.
$$

Since we assume that $\gamma_0^{\text{PTA}} = \gamma_0^{\text{non-PTA}}$ then we perform a grid search over different values for $a_0^{\text{PTA}}$ and $a_0^{\text{non-PTA}}$ to find the respective values such that $\gamma_0^{\text{PTA}} = \gamma_0^{\text{non-PTA}} \leq 1$. All the solutions imply a $\hat{\omega}_0$ above the mean. We then set the pre-GTC parameters to $\{\gamma_0^{\text{PTA}}, a_0^{\text{PTA}}, a_0^{\text{PTA}}\} = \{0.94, 1.20, 1.38\}$.

To recover the shocks to uncertainty parameters during the GTC we take the log average over four quarters of

$$
\tilde{U}_t = \left[ \frac{1 + \beta_t [\hat{\omega}_t (a_t a_0, r_t) - 1]}{1 + \beta_t [\hat{\omega}_t (a_t a_0, r_t) - 1]} \right]^{\frac{1}{\lambda}}.
$$

The LHS is recovered following the approach in C.2. In the RHS, we update $r_t = \{m_t, \Sigma_t\}$. Since we have two parameters and one expression, then we set $m_t^{\text{non-PTA}} = 1$ and recover the corresponding value for $\Sigma_t. 0.68$. After recovering $\Sigma_t$ for non-PTA we keep the assumption that this parameter is similar for PTA and non-PTA countries. This allow us to recover a $m_t^{\text{PTA}}$. Following the procedure just described and we get $m_t^{\text{PTA}} = 0.33$.

Appendix D. Supplementary data

A supplementary appendix can be found online at https://doi.org/10.1016/j.jinteco.2022.103661.

References

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