Bargaining with Incomplete Information

Lawrence M. Ausubel, Peter Cramton, and Raymond J. Deneckere*

June 2000

JEL No.: C78 (Bargaining Theory), D82 (Asymmetric and Private Information)
Keywords: Bargaining, sequential bargaining, incomplete information, asymmetric information, private information, Coase Conjecture

Send comments to:

Lawrence M. Ausubel
Department of Economics
University of Maryland
College Park, MD 20742-7211

ausubel@econ.umd.edu
301.405.3495

Peter Cramton
Department of Economics
University of Maryland
College Park, MD 20742-7211

peter@cramton.umd.edu
301.405.6987

Raymond J. Deneckere
Department of Economics
University of Wisconsin
Madison, WI 53706

rjdeneck@facstaff.wisc.edu
608.263.6724

*The authors gratefully acknowledge the support of National Science Foundation grants SBR–94–10545, SBR–94–22563, SBR–94–23104 and SBR–97–31025.

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1 Introduction

A central question in economics is understanding the difficulties parties have in reaching mutually beneficial agreements. Why do labor negotiations sometimes involve a strike by the union? Why do litigants engage in lengthy legal battles? And why does a worker with a grievance find it necessary to resort to a costly arbitration procedure? In all these cases, the parties would be better off if they could settle at the same terms without a protracted dispute. What, then, is preventing them from settling immediately? Recent theoretical work in economics has sought to answer this question.

Although the theory is still far from complete, researchers have taken promising steps in modeling bargaining disputes by focusing on the process of bargaining.[1] In the theory, costly disputes are explained by incomplete information about some aspect critical to reaching agreement, such as a party's reservation price.[2] Informational differences provide an appealing explanation for bargaining inefficiencies. If information relevant to the negotiation is privately held, the parties must learn about each other before they can identify suitable settlement terms. This learning is difficult because of incentives to misrepresent private information. Bargainers may have to engage in costly disputes to signal credibly the strength of their bargaining positions.

In this chapter, we provide an overview of the theoretical and empirical literature on bargaining under incomplete information. Since the literature on the topic is vast, it was inevitable that we had to limit the scope of our discussion. Consequently, a number of interesting and important contributions were left out. In particular, we would have liked to have had space to discuss the work on repeated bargaining (e.g., Hart and Tirole, 1988; Kennan, 1997; Vincent, 1998), and the extensive literature on durable goods monopoly (studying such topics as the impact of depreciation and increasing marginal cost of production, the effect of secondhand markets and transactions cost, and selling versus leasing contracts).

2 Mechanism Design

We begin with an analysis of the fundamental incentives inherent in bargaining under private information. For this, we abstract from the process of bargaining. Rather than model bargaining as a sequence of offers and counteroffers, we employ mechanism design and analyze bargaining mechanisms

[1] See Binmore, Osborne and Rubinstein (1992), Kennan and Wilson (1993), and Osborne and Rubinstein (1990) for surveys.

[2] Other motivations for disputes have been presented, such as uncertain commitments (Crawford, 1982) and multiple equilibria in the bargaining game (Fernandez and Glazer, 1991; Haller and Holden, 1990).

as mappings from the parties' private information to bargaining outcomes. This allows us to identify properties shared by all Bayesian equilibria of any bargaining game.

One basic question is whether private information prevents the bargainers from reaping all possible gains from trade. Myerson and Satterthwaite (1983) find that ex post efficiency is attainable if and only if it is common knowledge that gains from trade exist; that is, uncertainty about whether gains are possible necessarily prevents full efficiency. Our development of this result follows several papers in the implementation literature (Mookherjee and Reichelstein, 1992; Makowski and Mezzetti, 1994; Krishna and Perry, 1997; and, especially, Williams, 1999).

Consider an allocation problem with n agents. Agent i has a valuation $v_i(a,t_i)$ for the allocation $a \in A$ when its type is $t_i \in T_i$. An agent's type is private information. There is a status quo allocation, $\tilde{a}$, defining each agent's reservation utility. We normalize each v_i such that the reservation utility $v_i(\tilde{a},t_i)=0$. Utility for i is linear in its value and money: $u_i(a,t_i,x_i) = v_i(a,t_i) + x_i$, where x_i is the money transfer that i receives. A mechanism $\langle a,x \rangle$ determines an allocation $a(r)$ and a set of money transfers $x(r)$ based on the vector r of reported types. We wish to determine if it is possible to attain efficiency (for all t) by a mechanism that satisfies the agents' incentive and participation constraints. Let $U_i(r_i|t_i)$, $V_i(r_i|t_i)$, and $X_i(r_i)$ denote i's interim utility, valuation, and transfer when i reports r_i and the other agents honestly report t_{-i}:

$$\begin{aligned} U_i(r_i|t_i) &\equiv E_{t_{-i}}[u_i(a(r_i,t_{-i}),t_i,x_i(r_i,t_{-i}))] \\ V_i(r_i|t_i) &\equiv E_{t_{-i}}[v_i(a(r_i,t_{-i}),t_i)] \\ X_i(r_i) &\equiv E_{t_{-i}}[x_i(r_i,t_{-i})]. \end{aligned}$$

Then $U_i(r_i|t_i) = V_i(r_i|t_i) + X_i(r_i)$. Let $U_i(t_i) \equiv U_i(t_i|t_i)$. The mechanism is incentive compatible if honest reporting is a best response: $U_i(t_i|t_i) \geq U_i(r_i|t_i)$ for all $t_i,r_i \in T_i$. Assume that t_i has a positive density f_i on an interval support $[\underline{t}_i,\overline{t}_i]$, and that $V_i(r_i|t_i)$ is continuously differentiable. Then from the Envelope Theorem, incentive compatibility implies for almost every $t_i \in T_i$

$$\frac{dU_i(t_i)}{dt_i} = \frac{\partial U_i(r_i = t_i|t_i)}{\partial t_i} = \frac{\partial V_i(r_i = t_i|t_i)}{\partial t_i},$$

which by the Fundamental Theorem of Calculus implies

(IC) $$U_i(t_i) = U_i(\underline{t}_i) + \int_{\underline{t}_i}^{t_i} \frac{\partial V_i(r_i = \tau_i|\tau_i)}{\partial t_i} d\tau_i.$$

The important implication of (IC) is that once the allocation $a(t)$ is specified, an agent's interim utility in any incentive compatible mechanism that implements $a(t)$ is uniquely determined up to a constant.

Now consider the efficient allocation $a^*(t) \in \operatorname{argmax} \sum_i v_i(a,t_i)$, which maximizes the gains from trade. We know that the Groves mechanism implements the efficient allocation $a^*(\cdot)$ in dominant strategies. The Groves mechanism has transfers

$$x_i(t) = \sum_{j \neq i} v_j(a^*(t), t_j) - k_i(t_{-i}).$$

The second term, $k_i(t_{-i})$, is an arbitrary constant that does not distort the agent's incentives. Since the agent is concerned with its interim payoff, we can without loss of generality replace $k_i(t_{-i})$ with a single constant K_i for each agent that does not depend on the others' types. The first term provides the proper incentives. Ignoring the non-distorting constant, each agent gets the entire gains from trade. Hence, regardless of the reports of the others, honest reporting maximizes each agent's utility, since this yields the maximal gains from trade given the reports of the others. Honest reporting is a dominant strategy.

We will now develop necessary and sufficient conditions for the ex post efficient outcome to be Bayesian-implementable. Observe that a Groves mechanism automatically satisfies (IC), since it is incentive compatible. Moreover, if we vary the constants K_i, the Groves mechanisms span the set of all interim utilities that satisfy (IC) and achieve full efficiency. Thus, for any incentive-compatible and efficient mechanism, there exists a Groves mechanism that yields the same interim payoffs in dominant strategies; in checking whether efficiency can be achieved, we can simply focus on Groves mechanisms.

However, in order for efficiency to be attained in any unsubsidized mechanism where participation is voluntary, the additional requirements of (interim) individual rationality and (ex ante) budget balancing[3] must be met:

(IR) $$U_i(t_i) \geq 0 \text{, for all } i \text{ and for all } t_i \in T_i .$$

(BB) $$\sum_i E_t\left[x_i(t)\right] \leq 0 .$$

That is, no type of any agent is made worse off by participating, and the sum of the expected transfers is nonnegative. Given the preceding paragraph, efficiency is attainable if and only if there exists a Groves mechanism satisfying (IR) and (BB). In the "basic" Groves mechanism with $K_i = 0$, each of the n agents needs to be awarded the entire gains from trade, yet the gains from trade are created only once by the mechanism. Hence, the "basic" Groves mechanism generates an expected deficit, $\sum_i E_t\left[x_i(t)\right]$, equal to

[3] In general, ex ante budget balancing is justified if there is a risk-neutral mediator (or other financier) who can absorb the risk of ex post budget imbalances. In the absence of such a player, one may need to impose the stronger condition of ex post budget balancing, $\sum_i x_i(t) \leq 0$. However, in the current context, ex ante budget balancing is equivalent to ex post budget balancing. This is because all players are risk neutral and, hence, can jointly costlessly absorb the risk associated with ex-post budget imbalances (see also Cramton, Gibbons and Klemperer, 1987).

(n−1) times the expected gains from trade. In other words, the "basic" Groves mechanism satisfies (IR) [4] but violates (BB), whenever the expected gains from trade are positive. More general Groves mechanisms can try to finance the deficit by taxing the agents, but (IR) limits the magnitude of those taxes. Indeed, let $U_i^K(t_i)$ denote the interim utility of agent i in the Groves mechanism with taxes $K=(K_1,\ldots,K_n)$. Since $U_i^K=U_i^0+K_i$, the tax to agent i can be no greater than $K_i = \underline{U}_i$, where $\underline{U}_i \equiv \inf\left\{U_i^0(t_i) \mid t_i \in T_i\right\}$ is the interim utility of the worst-off type in the "basic" Groves mechanism. We therefore have:

THEOREM 1 (Williams, 1999): Under incentive compatibility (IC), individual rationality (IR) and budget balancing (BB), efficiency is attainable if and only if:

(E) $$(n-1)E_t[\Sigma_i\, v_i(a^*(t),t_i)] \le \Sigma_i\, \underline{U}_i .$$

We now apply Theorem 1 to prove the Myerson and Sattertherwaite result. In this case, there are two agents, a seller S and a buyer B bargaining over the exchange of a good. Each knows its own valuation for the good, but not that of the other. The seller's valuation s is drawn from a distribution with positive density on $[\underline{s},\bar{s}]$; the buyer's valuation b is drawn independently from a distribution with positive density on $[\underline{b},\bar{b}]$. If $\bar{s} \le \underline{b}$, it is common knowledge that gains from trade exist, and it is trivial to see that efficiency is attained by a single-price mechanism: trade for sure at a price $p \in [\bar{s}, \underline{b}]$. This is incentive compatible, since the outcome does not depend on the report, and it is individually rational, since each party receives a nonnegative payoff in every realization. We thus concentrate on the non-trivial case where $\bar{s} > \underline{b}$. The "basic" Groves mechanism has the following description: if $b > s$, trade occurs, the buyer pays s and the seller receives b, so that both get a payoff equaling $b - s$, the gains from trade; if $b \le s$, then trade does not occur and both get a payoff of 0. The interim payoff to an agent is the expected gains from trade given the agent's value. Since the expected gains from trade are decreasing in the seller's value and increasing in the buyer's value, the worst-off types are seller $\bar{s}$ and buyer $\underline{b}$. Hence,

$$\underline{U}_S = E_b[(b-\bar{s})1_{\{b>\bar{s}\}}]$$
$$\underline{U}_B = E_s[(\underline{b}-s)1_{\{\underline{b}>s\}}].$$

The deficit from the basic Groves mechanism is the expected gains from trade, which can be broken into four terms:

$$\begin{aligned}E[(b-s)1_{\{b>s\}}] = {} & E[(b-s)1_{\{b>s\}} \mid s>\underline{b}; b<\bar{s}]\Pr(s>\underline{b}; b<\bar{s}) \\ & + E[b-s \mid b>\bar{s}]\Pr(b>\bar{s}) \\ & + E[b-s \mid s<\underline{b}]\Pr(s<\underline{b}) \\ & - E[b-s \mid s<\underline{b}; b>\bar{s}]\Pr(s<\underline{b}; b>\bar{s}).\end{aligned}$$

[4] (IR) is satisfied, since $\sum_i v_i(a^*(t),t_i) \ge \sum_i v_i(\tilde{a},t_i) = 0$.

Since $\bar{s} > \underline{b}$, the first term is positive. Hence, (E) will be violated if the sum of the last three terms is at least as big as $\underline{U}_S + \underline{U}_B$. But

$$E[b - s | b > \bar{s}]\Pr(b > \bar{s}) - \underline{U}_S = E[\bar{s} - s | b > \bar{s}]\Pr(b > \bar{s})$$
$$E[b - s | s < \underline{b}]\Pr(s < \underline{b}) - \underline{U}_B = E[b - \underline{b} | s < \underline{b}]\Pr(s < \underline{b}),$$

so (E) is violated if

$$E[\bar{s} - s | b > \bar{s}]\Pr(b > \bar{s}) + E[b - \underline{b} | s < \underline{b}]\Pr(s < \underline{b}) - E[b - s | s < \underline{b}; b > \bar{s}]\Pr(s < \underline{b}; b > \bar{s}) \geq 0.$$

But this can be rewritten as

$$\begin{aligned} &E[\bar{s} - s | s < \underline{b}; b > \bar{s}]\Pr(s < \underline{b}; b > \bar{s}) \\ &+E[b - \underline{b} | s < \underline{b}; b > \bar{s}]\Pr(s < \underline{b}; b > \bar{s}) \\ &-E[b - s | s < \underline{b}; b > \bar{s}]\Pr(s < \underline{b}; b > \bar{s}) \\ &+E[\bar{s} - s | s \geq \underline{b}; b > \bar{s}]\Pr(s \geq \underline{b}; b > \bar{s}) \\ &+E[b - \underline{b} | s < \underline{b}; b \leq \bar{s}]\Pr(s < \underline{b}; b \leq \bar{s}) \geq 0. \end{aligned}$$

This follows, since the first three terms sum to $E[\bar{s} - \underline{b} | s < \underline{b}; b > \bar{s}]\Pr(s < \underline{b}; b > \bar{s}) \geq 0$, and the last two terms are both nonnegative. We thus have:

COROLLARY 1 (Myerson and Satterthwaite, 1983): If there is a positive probability of gains from trade (i.e., if $\bar{b} > \underline{s}$), but if it is not common knowledge that gains from trade exist (i.e., if $\bar{s} > \underline{b}$), then no incentive compatible, individually rational, budget balanced mechanism can be ex post efficient.

Whenever there is some uncertainty about whether trade is desirable, ex post efficient trade is impossible. For this reason, private information is a compelling explanation for the frequent occurrence of bargaining breakdowns or costly delay. Inefficiencies are a necessary consequence of the strong incentives for misrepresentation between bargainers with private information.

Myerson and Satterthwaite's result depends crucially on the uncertainty being about players' valuations. For example, if players were uncertain about their respective fixed costs of delaying agreement, or about each others' discount factors, efficiency can be achieved by having players trade at a price between their (known) valuations.[5] The Myerson-Satterthwaite result also depends on independent types and risk neutrality. For example, Gresik (1991a) and McAfee and Reny (1992) show that when

[5] For this reason, we will only study the outcome of dynamic trading processes when uncertainty is about players' valuations. Important contributions to extensive form bargaining when uncertainty is about players' fixed cost of bargaining include Perry (1986), Rubinstein (1985b), and Bikchandani (1992), and when uncertainty is about discount factors include Rubinstein (1985a) and Cho (1990b).

types are correlated efficient trade may be possible. Finally, it matters that the supports of the distributions of valuations are intervals (Matsuo, 1989).

Since ex post efficiency cannot be obtained, it is natural to ask how much of the gains from trade can be realized. Returning to the framework above of a single seller and single buyer with independent private values, then an allocation rule is simply the probability of trade as a function of the valuations: $p(s,b)$. We wish to find the allocation rule p that maximizes the expected gains from trade, subject to incentive compatibility and individual rationality. Suppose s is drawn from the distribution F with density f and b is drawn from the distribution G with density g. Myerson and Satterthwaite (1983) show that the optimal allocation rule p solves

$$\max_{p(\cdot,\cdot)} E[(b-s)p(s,b)] \quad \text{subject to}$$
$$\underline{U}_S + \underline{U}_B = E\left[\left(b - \frac{1-G(b)}{g(b)} - s - \frac{F(s)}{f(s)}\right)p(s,b)\right] \geq 0$$
$$E_b[p(s,b)] \text{ decreasing; } E_s[p(s,b)] \text{ increasing.}$$

The monotonicity constraints are necessary for incentive compatibility. The interim probability of trade is (weakly) decreasing in the seller's valuation and (weakly) increasing in the buyer's valuation. The first constraint is individual rationality (the worst-off types get a non-negative payoff) for a mechanism that satisfies (IC). Ignoring the monotonicity constraints, the Lagrangian is

$$\max_{p(\cdot,\cdot)} E[(d(b,\alpha) - c(s,\alpha))p(s,b)], \text{ where}$$
$$c(s,\alpha) = s + \alpha\frac{F(s)}{f(s)} \qquad d(b,\alpha) = b - \alpha\frac{1-G(b)}{g(b)}.$$

Hence, by pointwise optimization the maximizing allocation rule is

$$p^{\alpha}(s,b) = \begin{cases} 1 & \text{if } d(b,\alpha) > c(b,\alpha) \\ 0 & \text{if } d(b,\alpha) \leq c(b,\alpha), \end{cases}$$

where $\alpha \in (0,1]$ is chosen so that $\underline{U}_S + \underline{U}_B = 0$. A sufficient condition for the required monotonicity of the interim probability of trade is that $c(s,1)$ and $d(b,1)$ are increasing. This is the regular case.[6]

As an example, suppose both traders' valuations are drawn uniformly from [0,1]. Then $\alpha = 1/3$ and the optimal allocation rule is to trade if and only if the gains from trade $b - s$ is greater than 1/4. By surely trading when the gains from trade are largest, the mechanism reaps 84% of the possible gains from trade; there is a 16% loss due to the private information. The simultaneous-offer bargaining game studied by Chatterjee and Samuelson (1983) implements this optimal outcome. Both seller and buyer simultaneously

[6] Gresik (1991b) shows that we can replace interim individual rationality with the stronger ex post individual rationality without changing the set of ex ante efficient trading rules.

make offers. If the seller's offer is less than the buyer's, then they trade at a price half-way between the two offers. Otherwise, they do not trade. In the ex ante efficient equilibrium, the traders use the following linear strategies: the seller offers $2s/3 + 1/4$ and the buyer offers $2b/3 + 1/12$. In choosing offers, both recognize the fundamental tradeoff between the probability of trade and the terms of trade. Whenever the probability of trade is positive, the parties have an incentive to misrepresent: the seller overstates her value and the buyer understates. The size of the misrepresentation increases with the probability of trade.

Our derivation above assumed that the seller's private information does not affect the buyer's valuation for the object, and conversely that the buyer's private information does not affect how much the seller values the object. However, as emphasized by Akerlof (1970), there are many interesting trading situations in which traders' valuations are interdependent. A seller of a used car may have information about reliability relevant to a potential buyer, and the buyer of an oil tract may have survey information relevant to its seller. While dominant strategy mechanisms no longer exist when valuations are interdependent, several authors have recently constructed generalized Groves mechanisms for which efficient trade is a Bayesian equilibrium (Ausubel, 2002; Dasgupta and Maskin, 2000; Jehiel and Moldovanu, 2001; and Perry and Reny, 1998). These mechanisms could be used to derive an inefficiency result analogous to Myerson and Satterthwaite's (see Gresik, 1991c). Here we will consider the simpler environment studied by Akerlof, in which the seller's value s is private information, and the buyer's value is an increasing function of s satisfying $g(s) > s$. Note that the private values model is a special case in which $g(s)$ is constant at the level b. For this environment, Samuelson (1984) and Myerson (1985) established the following result:

THEOREM 2: A bargaining mechanism $\{p,x\}$ is incentive compatible and individually rational if and only if $p(\cdot)$ is weakly decreasing,

$$K \equiv \int_{\underline{s}}^{\overline{s}} \left(g(s) - s - \frac{F(s)}{f(s)} \right) f(s) p(s) ds \geq 0, \text{ and}$$

$$x(s) = k + sp(s) + \int_{s}^{\overline{s}} p(z)dz \text{, for some } 0 \leq k \leq K.$$

Note that, since $g(s) > s$, ex post efficiency requires that $p(s) \equiv 1$. Integrating the first inequality in Theorem 2 by parts, we see that this can be a trading outcome only if $E[g(s)] \geq \overline{s}$, i.e. the buyer's expected value exceeds the highest seller valuation. This condition is automatically satisfied in the private values case, but is restrictive in the interdependent case. In this sense, interdependencies in valuations make trading inefficiencies more likely. For example, if $g(s) = \beta s$ and s is uniform on [0,1], ex post efficiency requires $\beta \geq 2$.

Akerlof went one step further, and observed that adverse selection in the above model may be so severe that no market-clearing price involving a positive level of trade can exist. This happens whenever $\mathrm{E}[g(v) - s \mid v \leq s] < 0$ for all $s > \underline{s}$, for then any price that all seller types below s would accept yields the buyer negative expected surplus. Akerlof only considered single-price mechanisms, and it is of course conceivable that under his condition some more general trading mechanism could prove superior to competitive equilibrium. However, it is possible to use Theorem 2 to show that this cannot happen: under Akerlof's condition, the only incentive-compatible mechanism is the zero-trade mechanism. We can again illustrate this with the linear example described above; since $\mathrm{E}[\beta v - s \mid v \leq s] = (\beta/2 - 1)\, s^2$, Akerlof's condition reduces to $\beta < 2$. It follows that $g(s) - s - F(s) / f(s) = (\beta - 2)\, s < 0$, so the incentive compatibility condition $K \geq 0$ can be satisfied only if $p(s) = 0$.

An important generalization of the bilateral independent values model is to multiple sellers and buyers. How does the bargaining inefficiency change as we add traders? Rustichini, Satterthwaite, and Williams (1994) consider a model with m sellers and m buyers and price is set to equate revealed demand and supply. In any equilibrium, the amount by which a trader misreports is $O(1/m)$ and the inefficiency is $O(1/m^2)$.[7] Hence, the inefficiency caused by private information quickly falls toward zero as competition increases. This provides a justification for assuming full information in competitive markets.

The mechanism design approach does not just apply to static trading procedures. Indeed, if the traders discounts by the same interest rate r, then all the results above generalize to dynamic trading mechanisms, where the probability of trade $p(s,b)$ is replaced with the time of trade $t(s,b)$, where $p(s,b) = e^{-r\,t(s,b)}$. Hence, ex post efficiency is unobtainable as a Bayesian equilibrium in any static or dynamic bargaining game when it is uncertain whether trade is desirable.

An important feature of the ex ante efficient trading rule is that it is static. Trade either occurs immediately or not at all. Such static trading rules have been criticized, because they violate sequential rationality (Cramton, 1985). Their implementation requires a commitment to walk away from know gains from trade. For example, in the Chatterjee-Samuelson mechanism, with probability 7/32, the offers reveal that the gain from trade is positive, but less than 1/4, so the parties are required not to trade, even though both know that mutually beneficial trade is possible. In addition, with probability 7/16, at least one trader knows that both are sure to get 0 in the mechanism. This provides an incentive to propose another trading

[7] See also Gresik and Satterthwaite (1989), Satterthwaite and Williams (1989), Williams (1990, 1991), and Wilson (1985).

rule, even before offers are announced. An initial round of "cheap talk" may upset the equilibrium (Farrell and Gibbons 1989).

Cramton, Gibbons, and Klemperer (CGK) (1987) generalize the Myerson and Satterthwaite (MS) problem to the case of n traders who share in the ownership of a single asset. Specifically, each trader $i \in \{1,...,n\}$ owns a share $r_i \geq 0$ of the asset, where $r_1 + \ldots + r_n = 1$. As in MS, player i's valuation for the entire good is v_i, and the utility from owning a share r_i is $r_i v_i$, measured in monetary terms. The v_i's are independent and identically distributed according to F with positive density f on $[\underline{v}, \overline{v}]$. A partnership (r,F) is fully described by the vector of ownership rights $r = \{r_1,...,r_n\}$ and the traders' beliefs F about valuations.

MS consider the case $n = 2$ and $r = \{1,0\}$. They show that there does not exist a Bayesian equilibrium of the trading game that is individually rational and ex post efficient. In contrast, CGK show that if the ownership shares are not too unequally distributed, then it is possible to satisfy both individual rationality and ex post efficiency.

In addition to exploring the MS impossibility result, this paper considers the dissolution of partnerships, broadly construed. In a situation of joint ownership, who should buy out whom and at what price? Applications include divorce and estate fair-division problems (McAfee, 1992), and also public choice. For example, when several towns jointly need a hazardous-waste dump, which town should provide the site and how should it be compensated by the others?

In this context, ex post efficiency means giving the entire good to the partner with the highest valuation. A partnership (r,F) can be dissolved efficiently if there exists a Bayesian equilibrium of a Bayesian trading game that is individually rational and ex post efficient.

THEOREM 3. The partnership (r,F) can be dissolved efficiently if and only if

$$\text{(D)} \qquad \sum_{i=1}^{n}\left[\int_{v_i^*}^{\overline{v}}[1-F(u)]\,u\,dG(u)-\int_{\underline{v}}^{v_i^*}F(u)\,u\,dG(u)\right]\geq 0,$$

where v_i^* solves $F(v_i)^{n-1} = r_i$ and $G(u) = F(u)^{n-1}$.

Equation (D) is equivalent to (E) applied to this setting. As an example, if $n = 2$ and values are uniformly distributed on [0,1], then the partnership is dissolvable if and only if no shareholder's share is larger than .789. In general, the set of dissolvable partnerships is a convex, symmetric subset of the unit simplex centered at equal shares.

COROLLARY 2. For any distribution F, the one-owner partnership $r = \{1,0,...,0\}$ cannot be dissolved efficiently.

This corollary generalizes the MS impossibility result to the case of many buyers. The one-owner partnership can be interpreted as an auction. Ex post efficiency is unattainable because the seller's reservation value v_1 is private information. The seller finds it in her best interest to set a reserve above her value v_1. The corollary also speaks to the time-honored tradition of solving complex allocation problems by resorting to lotteries: even if the winner is allowed to resell the object, such a scheme is inefficient because the one-owner partnership that results from the lottery cannot be dissolved efficiently.

CGK demonstrate that the incentives for misrepresentation depend on the ownership structure. The extreme 0-1 ownership shares in bilateral bargaining maximize the incentive for misrepresentation: sellers have a clear incentive to overstate value and buyers have a clear incentive to understate. Partial ownership introduces countervailing incentives, since the parties no longer are certain whether they are buying or selling. In the case of bilateral bargaining, the worst-off types are the highest seller type and the lowest buyer type. These trader types are unable to misrepresent (a seller cannot claim to have a value greater than $\bar{s}$ and a buyer cannot claim to have a value less than $\underline{b}$); hence, these types need not receive any information rents. With partial ownership r_i, the worst-off type is v_i^*, which solves $F(v_i)^{n-1} = r_i$. Notice that $r_i = F(v_i^*)^{n-1}$ is the probability that type v_i^* has the highest value and thus buys $1 - r_i$ of the good in the ex post efficient mechanism. Likewise, with probability $1 - r_i$, type v_i^* sells r_i. Hence, for the worst-off type, the expected purchases, $r_i(1 - r_i)$, equal the expected sales, $(1 - r_i)r_i$. In this sense, the worst-off type is the most confused about whether she is buying or selling; the incentives to overstate just balance the incentives to understate, and no bribes are required to get the trader to report the truth.

A basic insight of this analysis is that when parties have private information, bargaining efficiency depends on the assignment of property rights (see also Samuelson, 1985; and Ayres and Tally, 1994). Hence, full information is an essential ingredient in the Coase (1960) Theorem that bargaining efficiency is not affected by the assignment of property rights.

Mechanism design is a powerful theory for studying incentive problems in bargaining. We are able to characterize the set of outcomes that are attainable, recognizing each trader's voluntary participation and incentive to misrepresent private information. In addition, we are able to determine optimal trading mechanisms—mechanisms that are efficient in an ex ante (or interim) sense. Despite these virtues, mechanism design has two weaknesses. First, the mechanisms depend in complex ways on the traders' beliefs and utility functions, which are assumed to be common knowledge. Second, it allows too much commitment. In practice, bargainers use simple trading rules—such as a sequence of offers and counteroffers—that do not depend on beliefs or utility functions. And bargainers may be unable to walk away from known gains from trade. For this reason we next turn to the analysis of particular dynamic bargaining games.

3 Sequential Bargaining with One-Sided Incomplete Information: The "Gap" Case

In the previous section, we described bargaining as being static and mediated. Instead, we will now assume that bargaining occurs through a dynamic process of bilateral negotiation. A bargaining protocol explicitly specifies the rules that govern the negotiation process, and the bargaining outcome is described as an equilibrium of this extensive-form game.

We follow Rubinstein (1982) in requiring that only one offer can be on the bargaining table at any one time,[8] and that once an offer is rejected it becomes void (i.e., does not constrain any player's future acceptance or offer behavior). More precisely, we assume that there are an infinite number of time periods, denoted by $n = 0,1,2,...$. In each period in which bargaining has not yet concluded, one of the players (whose identity is a function only of the time period n) can make an offer to his bargaining partner consisting of a price $p \in \mathbb{R}$ at which trade is to occur. Upon observing this offer, the partner can either accept, in which case the object is exchanged at the specified price and the bargaining ends, or reject, in which case the play moves on to the next period. Note that any terminal node of the game is uniquely identified by a pair (p,n). We assume that players are impatient, discounting surplus at the common discount factor $\delta \in [0,1)$. Hence the payoffs assigned to terminal node (p,n) are $\delta^n(b-p)$ and $\delta^n(p-s)$, for the buyer and seller, respectively.

Three bargaining protocols of this type will be of specific interest: the *seller-offer game*, in which only the seller is allowed to make offers; the *alternating-offer game*, in which the buyer and seller alternate in making proposals; and the *buyer-offer game*, in which the buyer makes all the offers.

The private information is modeled as follows. Before the bargaining begins (i.e., prior to period 0), nature selects a signal $q \in [0,1]$, and informs one of the two parties of its realization. The distribution of the signal is common knowledge and, without loss of generality, will be assumed to be uniform. The signal in turn determines the buyer and seller valuations through the monotone functions $v(\bullet)$ and $c(\bullet)$:

$$b = v(q) \qquad s = c(q).$$

We will say that the model has *private values*, if the uninformed party's valuation function is constant, and that the model has *interdependent values*, otherwise. We will adopt the convention that if the buyer is the informed party then the function $v(q)$ is decreasing, so that it represents an (inverse)

[8] It is well known that even in the one-shot complete-information case simultaneous offers permit any outcome. See also Sákovics (1993) for an illuminating discussion on the importance of precluding simultaneous offers.

demand function, and if the seller is the informed party then the function $c(q)$ is increasing, so that it represents an (inverse) supply function. The signal q is thus just an index indicating the rank order of the types of the informed party. Throughout, it will be assumed that the functions $v(\bullet)$ and $c(\bullet)$ are common knowledge.

Note that, in every period n, the information set of the offering player can be identified with a history of n rejected offers, and the information set of the receiving player can be identified with the same history concatenated with the current offer. For the offering player, a pure behavioral strategy in period n specifies the current offer as a function of this history of rejected offers. For the player receiving an offer, a pure behavioral strategy in period n specifies a decision in the set {A,R} as a function of the n-history of rejected offers and the current offer (where A denotes acceptance and R denotes rejection of the current offer). A sequential equilibrium consists of a pair of behavioral strategies and a system of beliefs. Specifically, a sequential equilibrium associates with every node at which it is the uninformed party's turn to move a belief over the signal (rank order) of the informed party. As indicated above, the initial belief is that q is uniform on [0,1]. Sequential equilibrium requires that the beliefs are "consistent", i.e., are updated from the belief in the previous period and the equilibrium strategies using Bayes' law (whenever it is applicable). Sequential equilibrium also requires that each player's strategy be optimal after any history, given the current beliefs.

Offer/counteroffer bargaining games typically have a plethora of equilibria, for two distinct reasons. First, somewhat analogous to the folk-theorem literature in repeated games, the presence of an infinite number of bargaining rounds permits history-dependent strategies that can often support a wide variety of equilibrium behavior (Ausubel and Deneckere, 1989a,b). Secondly, even if bargaining were allowed to last only a finite number of periods, there will typically still exist a multiplicity of sequential equilibria. This multiplicity arises because sequential equilibrium imposes no restrictions on players' beliefs following out-of-equilibrium moves (Bayes' law is then simply not applicable). As a consequence, an out-of-equilibrium offer by the informed party can lead to adverse inferences regarding its eagerness to conclude the transaction, resulting in poor terms of trade. In alternating-offer bargaining games, the threat of such adverse inferences can therefore often sustain a wide variety of bargaining outcomes (Fudenberg and Tirole, 1983; Rubinstein, 1985a,b).

In order to narrow down the range of predicted bargaining outcomes, researchers have confined attention to more restrictive equilibrium notions. One refinement that has received considerable attention is the concept of stationary equilibrium (Gul, Sonnenschein and Wilson, 1986). Recall that a belief is a probability distribution $F(q)$ over the set of possible signals (the unit interval). We will say that a belief $G(q)$ is a *truncation* (from the left) of the belief $F(q)$ if it is the conditional probability distribution derived from $F(q)$, given that the signal exceeds some threshold level $q' > 0$. Thus $G(q) = 0$ for $q < q'$ and $G(q) = [F(q) - F(q')]/[1 - F(q')]$ for $q \geq q'$. A *stationary equilibrium* is a sequential equilibrium satisfying three additional conditions:

(1) Along the equilibrium path, the beliefs following rejection of the informed party's offer are a truncation of the beliefs entering that period;

(2) For every history such that the current beliefs are a truncation of the priors, the informed party's current acceptance behavior is a function only of the current offer; and

(3) For every history such that the current belief is the same truncation of the prior, the informed party's current offer behavior is identical.

The notion of stationarity is rather subtle, and to understand its meaning it is useful to first restrict attention to the game in which the uninformed party makes all the offers, so that only requirement (2) carries any force. Observe that, in any offer/counteroffer game, rejections by the informed party always lead to a truncation of the current beliefs:[9]

LEMMA 1 (Fudenberg, Levine and Tirole, 1985): Let n be a period in which it is the uninformed party's turn to make an offer, and denote the history of rejected prices entering period n by h_n. Then to every sequential equilibrium there corresponds a nonincreasing (nondecreasing) function $P(h_n,q)$ and equivalent sequential equilibrium such that if the informed party is the buyer (seller), it accepts the current offer p if and only if $p \leq P(h_n,q)$ (respectively, $p \geq P(h_n,q)$).

PROOF: Suppose buyer type q is willing to reject the current offer p. Any buyer type $q' > q$ can always mimic the strategy of type q, and thereby secure the same expected probability of trade and expected payment from rejecting p. The single crossing property then implies that if $v(q') < v(q)$, type q' will strictly prefer rejection to acceptance. Meanwhile, if q is indifferent between accepting and rejecting, a purification argument shows that there is an equivalent sequential equilibrium and a cutoff signal level q'' with $v(q'') = v(q)$, such that all $q' < q''$ accept p and all $q' > q''$ reject p. ■

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[9] Sequential equilibria of the game in which the uninformed party makes all the offers therefore have a screening structure, with higher valuation buyer types trading earlier and at higher prices than lower valuation types. Delaying agreement by rejecting the current offer credibly signals to the seller that the buyer has a lower valuation, thereby making her willing to lower price over time.

For the game where the uninformed party makes all the offers, Lemma 1 implies that the informed party uses a possibly history-dependent reservation price strategy, $P(h_n,q)$. Requirement (2) in the definition of stationarity requires that the acceptance functions $P(h_n,q)$ are constant over all histories h_n. It is this history independence that gives stationarity its cutting power. Stationarity is a stronger restriction than Markov-perfection (Maskin and Tirole, 1994), since the latter would only require that P be constant on histories inducing the same current beliefs. As emphasized by Gul and Sonnenschein (1988) stationarity also embodies a form of monotonicity: when the uninformed party is more optimistic (in the sense that the beliefs are truncated at lower level), the informed party must not be tougher in its acceptance behavior.

For game structures that permit the informed party to make offers, stationarity carries two additional restrictions. The informed party's offer behavior must be Markovian (requirement #3); and in equilibrium the beliefs following a period in which the informed party made an offer must be a truncation of the prior (requirement #1). Thus, stationarity imposes a screening structure on the equilibrium. This assumption is very strong, since it requires the uninformed party to accept with probability zero or one following any equilibrium offer that is not made by all types, and thereby severely restricts the informed party's ability to signal its type. At the same time, however, stationarity may be insufficiently restrictive because it does not address the multiplicity of equilibria arising from "threatening with beliefs." Furthermore, refinements of sequential equilibrium designed to reduce this multiplicity are potentially at odds with the requirements of stationarity. This raises the question of whether stationary equilibria (with or without additional refinements) are always guaranteed to exist. Fortunately, as we shall see, the answer to this question is broadly positive.

In the remainder of this section, we study the trading situation in which it is common knowledge that the gains from trade are bounded away from zero, i.e., there exists $\Delta > 0$ such that $v(q) - c(q) \geq \Delta$ for all $q \in [0,1]$. Section 4 studies the case where there is no such Δ, so that the gains from trade can be arbitrarily small.

3.1 Private Values

To facilitate the discussion of private values model, we will henceforth assume that the informed party is the buyer (the symmetric situation in which it is the seller that is informed is treated in the subsection on interdependent values). In this case, the seller's cost is independent of the signal level and can without loss of generality be normalized to zero (by measuring buyer valuations net of cost). The model is therefore completely described by the discount factor δ and the nonincreasing buyer valuation function $v(q)$. In order to permit the existence of an equilibrium, $v(q)$ will be assumed to be left continuous (to see this is necessary, consider the seller-offer game in which $\delta = 0$).

3.1.1 The Seller-Offer Game

Following Fudenberg, Levine and Tirole (1985) and Gul, Sonnenschein and Wilson (1986), we are interested in stationary equilibria in which the buyer's acceptance behavior depends upon previous history only to the extent it is reflected in the current price. The purification argument in the proof of Lemma 1 shows that there is no loss of generality in assuming that the buyer does not randomize in his acceptance behavior, an assumption which we will maintain henceforth. The buyer's acceptance behavior is thus completely characterized by a nonincreasing (left-continuous) acceptance function $P(q)$. Consequently, following any history the seller's belief will always be a truncation of the prior, i.e., be uniform on an interval of the form $[Q,1]$. The lower endpoint of this interval, Q, is thus a *state variable*.

The acceptance function acts as a static demand curve for the seller, who faces a tradeoff between screening more finely and delaying agreement. This tradeoff is captured by the dynamic programming equation:

(1) $$W(Q) = \max_{Q' \geq Q} \left\{ P(Q') \frac{(Q'-Q)}{(1-Q)} + \delta \frac{(1-Q')}{(1-Q)} W(Q') \right\}.$$

To understand (1), observe that if the seller brings the state to Q' (by charging the price $P(Q')$), then the buyer will accept with conditional probability $(Q'-Q)/(1-Q)$. Rejection happens with complementary probability, moves the state to Q', and results in the seller receiving the value $W(Q')$ with a one-period delay. Letting $V(Q) = (1-Q)\, W(Q)$ denote the seller's ex-ante expected value from trading with buyer types in the interval $(Q,1]$, equation (1) can be simplified to:

(2) $$V(Q) = \max_{Q' \geq Q} \left\{ P(Q')\,(Q'-Q) + \delta\, V(Q') \right\}.$$

Let $T(Q)$ denote the argmax correspondence in (2). By the generalized Theorem of the Maximum (Ausubel and Deneckere, 1993b), T is nonempty and compact-valued, and the value function V is continuous. A straightforward revealed preference argument also shows that T is a nondecreasing correspondence, and hence single-valued at all but at most a countable set of Q.

Define $t(Q) = \min T(Q)$, and note that $t(Q)$ is continuous at any point where $T(q)$ is single-valued.

Now consider any point Q where $v(\bullet)$, $P(\bullet)$ and $t(\bullet)$ are continuous; consumer optimization then requires that:

$$(3) \qquad P(Q) = (1-\delta)\, v(Q) + \delta\, P(t(Q)) \;.$$

Equation (3) says that when the seller charges the price $p = P(Q)$, the buyer of type $q = Q$ must be indifferent between accepting the offer p, and waiting one period to accept the next offer (which must be $P(t(Q))$. A straightforward argument establishes that the consumer indifference equation (3) must in fact hold for all $Q > 0$.[10] This fact has an important consequence: in any stationary equilibrium, *the seller will never randomize except (possibly) in the initial period*.[11] Indeed, in period zero the seller is free to randomize amongst any element of $T(0)$. However, given any such choice Q, equation (3) requires the seller to select $t(Q)$ in the next period (even if $T(Q)$ is not single-valued). This is necessary to make the buyer's acceptance decision optimal.

The triplet $\{P(\bullet),V(\bullet),t(\bullet)\}$ completely describes a stationary equilibrium. After any history in which the seller selects a price $p = P(Q)$ for some Q, all consumer types $q \leq Q$ accept and all others reject; the next period the seller lowers the price to $P(t(Q))$. If the seller were ever to select a price p such that $\sup\{P(Q') : Q' > Q\} < p < P(Q)$ for some Q, then the highest consumer type to accept is again Q. However, if the gap in the range of P is due to a discontinuity in the function $t(Q)$, then to make consumer Q's acceptance rational, the seller must in the next period randomize between the offers in $P(T(Q))$ so as to make Q indifferent. Note, however, that an optimizing seller will never charge a price in this range, as she could induce exactly the same the same set of buyer types to accept by charging the higher price $P(Q)$. Randomization is therefore only called for if the seller made a mistake in the previous period.

Any stationary equilibrium path has the following structure. In the initial period, the seller selects (possibly randomly) a price $P(Q_0)$, for some $Q_0 \in T(0)$. Note that randomization is possible only if $T(0)$ is multiple valued, i.e., its profit function has multiple maximizers. This should be a rare occurrence, because as a monotone correspondence, $T(Q)$ can have at most countably many points at which it is not

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[10] Consider any of the (at most countably many) excluded states Q, and let $\{Q_n\}$ be a sequence of nonexcluded points converging from below to Q. Since (3) holds for each n, upon taking limits as $n \to \infty$, we see that (3) holds for all $Q > 0$.

[11] Gul, Sonnenschein and Wilson (1986, Theorem 1) constructively demonstrate the absence of randomization along the equilibrium path, under the assumption that there is a gap and condition (L) of theorem 4 (below) holds. The argument given here (drawn from Ausubel and Deneckere, 1989a, Proposition 4.3) shows that it is stationarity that is the driving force behind this result.

single valued (see the genericity statement in Theorem 4, below). The remainder of the future is then entirely deterministic, with the seller successively lowering the prices to $P(t(Q_0))$, $P(t^2(Q_0))$, $P(t^3(Q_0))$, ... , and corresponding buyer acceptances in $(Q_0,t(Q_0)]$, $(t(Q_0),t^2(Q_0)]$, $(t^2(Q_0), t^3(Q_0)]$, ...

An important question is whether the coupled pair of functional equations (2) and (3) has a solution. At the same time, the bootstrap structure of these equations suggests that there may be a severe multiplicity of stationary triplets. The pioneering work in the areas of existence and uniqueness of stationary equilibria is due to Fudenberg, Levine and Tirole (1985) and Gul, Sonnenschein and Wilson (1986). Below, we collect a number of disparate results in the literature into a single theorem:

THEOREM 4: For any left-continuous valuation function $v(\bullet)$, there exists a stationary equilibrium of the seller-offer game. Every stationary equilibrium is supported by a stationary triplet $\{P,t,V\}$ satisfying (2) and (3). Furthermore, if there is a gap, and if the demand curve satisfies a Lipschitz condition at $q = 1$:

(L) There exists $L < \infty$ such that $v(q) - v(1) \leq L(1-q)$, for all $q \in [0,1]$,

then the stationary triplet is unique, every sequential equilibrium outcome coincides with a stationary equilibrium outcome, and for generic values of the state there is a unique stationary (and hence sequential) equilibrium outcome. Under these conditions, there also exists a finite integer $\hat{N}$ such that the buyer accepts the seller's offer by period $\hat{N}$, regardless of the discount factor δ.

PROOF: Fudenberg, Levine and Tirole (1985, Propositions 1 and 2) prove existence and generic uniqueness of the outcome path in the case of a gap, under the assumption that the demand curve is differentiable with derivative bounded above and below. Gul, Sonnenschein and Wilson (1986, Theorem 1) prove existence and uniqueness of a stationary triplet when there is a gap and condition (L) holds, and also demonstrate generic uniqueness of the outcome path. A general existence proof appears in Ausubel and Deneckere (1989a, Theorem 4.2). Deneckere (1992) proves that under condition (L) the number of bargaining rounds is uniformly bounded for fixed $v(\bullet)$. ■

To make matters more concrete, and also to illustrate some of the ideas behind Theorem 4, let us work out a simple example in which the buyer's valuation can take on two possible values, $\overline{b} > \underline{b} > 0$:

$$(4) \qquad v(q) = \begin{cases} \overline{b} & 0 \leq q \leq \hat{q} \;, \\ \underline{b} & \hat{q} < q \leq 1 \;. \end{cases}$$

Note that this example is in the case of a "gap" and satisfies condition (L), so by Theorem 4 there exists a unique stationary triplet.

First, let us consider the case where $\hat{q}\,\bar{b} < \underline{b}$, i.e., the monopoly price on the static demand curve (4) equals $\underline{b}$. Observe that since the seller will never offer a price more favorable than she would if she were facing the strongest buyer type for sure, the buyer will always accept any price below $\underline{b}$ with probability one. Thus, in any sequential equilibrium, the seller's payoff must be no lower than her static monopoly profits, $\underline{b}$. Meanwhile, Stokey (1979) showed that the optimal selling policy of a dynamic monopolist with perfect commitment power consists of charging the static monopoly price, and never lowering price thereafter (see also the closely related "no-haggling" result of Riley and Zeckhauser, 1983). Since a monopolist lacking commitment power can only do worse, the seller's equilibrium profits must also be no higher than her static equilibrium profits. We conclude that there is a unique sequential equilibrium outcome, with the seller charging the price $\underline{b}$, and all buyer types accepting. Note that this equilibrium is supported by the unique stationary triplet $V(Q) = (1-Q)\underline{b}$, $t(Q) = 1$, and using (3), $P(q) = (1-\delta)\bar{b} + \delta\,\underline{b}$ for $q \in [0,\hat{q}]$ and $P(q) = \underline{b}$ for $q \in (\hat{q},1]$.

When $\hat{q}\bar{b} > \underline{b}$, bargaining necessarily takes place over multiple periods, but the above argument still contains the key to uniqueness of the stationary triplet. Indeed, let us define q_1 as the lowest value of the state such that $\underline{b}$ is a monopoly price on the residual demand curve starting at q_1, i.e., $\bar{b}\,(\hat{q}-q_1) = \underline{b}\,(1-q_1)$. A parallel argument to the one given above then establishes that once the state reaches beyond q_1, the seller will *necessarily* end the bargaining immediately, by offering the price $\underline{b}$. The role of condition (L) in Theorem 4 is to more generally guarantee the existence of a critical level $q_1 < 1$ such that whenever the state exceeds q_1 the dispersion of valuations of the remaining buyer types is such that it no longer pays the seller to price discriminate amongst them.

With the endplay tied down, backward induction on the state then completes the uniqueness argument. To see how this works, observe that there exists a $q_2 < q_1$, such that whenever the state is in $(q_2,q_1]$ the seller will select to bring the state in the interval $(q_1,1]$. Indeed, whenever q_2 is sufficiently near q_1, any potential gain from increased price discrimination over the interval $(q_2,q_1]$ is outweighed by the loss due to delayed receipt of the profits $V(q_1)$. In our two-type model, when the state is in $(q_2,q_1]$ the monopolist will therefore offer $p_1 = (1-\delta)\bar{b} + \delta\,\underline{b}$, which all buyer types in $[0,\hat{q}]$ accept. In this fashion, we can keep on recursively extending the stationary triplet to the entire interval [0,1]. Buyer types in the interval $(q_i, q_{i-1}]$ will be indifferent between accepting p_i and waiting one period to receive p_{i-1}, and the state q_i is such that the monopolist is indifferent between offering p_i

(with all buyer types in $(q_i,q_{i-1}]$ accepting) and offering p_{i-1} (with all buyer types in $(q_i,q_{i-2}]$ accepting. More precisely, we can compute the following explicit solution.

Let $q_{-1} = 1$, $q_0 = \hat{q}$, and inductively define the sequence $q_1 > q_2 > ... > q_N$ from:

(5) $$m_n = \alpha\delta^{-(n-1)}m_{n-1} \qquad (n \geq 2) ,$$

and the initial condition $m_1 = (\alpha-1)\, m_0$, using $m_n = q_{n-1} - q_n$, $\alpha = \bar{b}/(\bar{b} - \underline{b})$, and $N = \min \{n : q_n \leq 0\}$. Also, let p_n be such that a buyer with valuation $\bar{b}$ is indifferent between accepting p_n today and waiting n periods to receive the offer $\underline{b}$:

(6) $$p_n = (1-\delta^n)\, \bar{b} + \delta^n\, \underline{b} .$$

THEOREM 5: Let $v(q)$ be given by (4), and let $q_N \leq 0 < q_{N-1} <...< q_0 = \hat{q}$ be defined by (5). Then with every (purified) sequential equilibrium of the seller-offer game is associated the unique stationary triplet:

$$P(Q) = p_n \qquad Q \in (q_n,q_{n-1}] ,$$

$$t(Q) = q_{n-2} \qquad Q \in (q_n,q_{n-1}] \text{ if } n > 1, \text{ and } Q \in (q_1,1] \text{ if } n=1 ,$$

$$V(Q) = p_{n-1}\, (q_{n-2}-Q)+\delta\, V(q_{n-2}) \qquad Q \in (q_n,q_{n-1}] \text{ if } n > 1, \text{ and } Q \in (q_1,1] \text{ if } n=1 .$$

PROOF: See Deneckere (1992).

According to Theorem 5, when $q_N < 0$ bargaining lasts for N periods. The seller starts out by offering the price $p_{N-1} = P(q_{N-2})$, which is accepted by all buyer types in the interval $[0,q_{N-2}]$. Play then continues with the seller offering p_{N-2}, which all buyer types in $(q_{N-2},q_{N-3}]$ accept, and so on, until the state q_0 is reached at which point the seller makes the final offer p_0. When $q_N = 0$, the seller can freely randomize between charging p_N and p_{N-1}. However, given the outcome of the randomization, the remainder of the equilibrium path is uniquely determined: if the seller initially selects p_N play lasts for $(N+1)$ periods, and if she selects p_{N-1} play lasts for N periods. Note, however, that the condition $q_N = 0$ is highly nongeneric, in two senses. First, if the initial state is slightly different from q_N the outcome is unique. Secondly, since the condition $q_N = 0$ is equivalent to $m_0 + ... + m_N = 1$, it follows from (5) that for generic (α,δ) the outcome path is unique.

The closed form (5) also allows us to investigate the behavior of the solution as bargaining frictions become smaller, i.e., players become more patient (see also Hart, 1989, Proposition 2). Intuitively, for fixed acceptance function P, the seller will discriminate more and more finely as she becomes more patient, approaching perfect price discrimination on the acceptance function P as δ converges to one. Counteracting this is that for fixed seller behavior, as the buyer becomes more patient, the acceptance function will become flatter and flatter, in the limit approaching the constant $\underline{b} = v(1)$ as δ converges to 1. If we fix δ_S and let δ_B converge to one, the seller loses all bargaining power. On the other hand, if we fix δ_B and let δ_S increase, the seller will gain bargaining strength (Sobel and Takahashi, 1983). With equal discount factors, the two forces more or less balance each other out. To see this, note from (5) that m_n is decreasing, and hence that the number of bargaining rounds N is increasing in δ. However, as the limiting solution to (5) is given by $m_n = \alpha^n m_0$, we see that regardless of the discount factor, the number of bargaining rounds is bounded above by:

$$\hat{N} = \min \{ n : \alpha^n m_0 \geq 1 \} .$$

While the number of equilibrium bargaining rounds therefore increases with δ, the existence of a uniform upper bound to the number of bargaining rounds implies that the cost of delay (as measured by the forgone surplus) vanishes as δ approaches one.

A slightly weaker, but qualitatively similar, proposition has become known in the literature as the "Coase Conjecture," after Nobel laureate Ronald Coase, who argued that a durable goods monopolist selling an infinitely-durable good to a demand curve of atomistic buyers would lose its monopoly power if it could make frequent price offers (Coase, 1972). The connection with the durable goods literature obtains because to every actual buyer type in the durable goods model, there corresponds an equivalent potential buyer type in the bargaining model. To formally state the Coase Conjecture, let us denote the length of the period between successive seller offers by z, and let r be the discount rate common to the bargaining parties, so that $\delta = e^{-rz}$. We then have:

THEOREM 6 (Coase Conjecture): Suppose we are in the case of a gap. Then for every $\varepsilon > 0$ and valuation function $v(\bullet)$, there exists $\bar{z} > 0$ such that, for every time interval $z \in (0,\bar{z})$ between offers and for every sequential equilibrium, the initial offer in the seller-offer bargaining game is no more than $\underline{b} + \varepsilon$ and the buyer accepts the seller's offer with probability one by time ε.

PROOF: Gul, Sonnenschein and Wilson (1986, Theorem 3).

Note that Theorem 6 immediately follows from Theorem 4, by selecting $\bar{z} \leq \varepsilon/\hat{N}$, and by noting that since the highest valuation buyer always has the option to wait until period $\hat{N}$ to accept the price $\underline{b}$, the seller's initial price can be no more than $(1 - \delta^{\hat{N}})v(0) + \delta^{\hat{N}}\underline{b}$, which converges to $\underline{b}$ as z converges to zero. For empirical or experimental work, Theorem 6 has the unfortunate implication that real bargaining delays can only be explained by either exogenous limitations on the frequency with which bargaining partners can make offers, or by significant differences in the relative degree of impatience between the bargaining parties.

3.1.2 Alternating Offers

When the uninformed party makes all the offers, the informed party has very limited means of communication. At any point in time, buyer types can only separate into two groups, those who accept the current offer and thereby terminate the game, and those who reject the current offer in order to trade at more favorable terms in the future. Since higher valuation buyer types stand to lose more from delaying trade, the equilibrium necessarily has a screening structure. In the alternating-offer game, screening will still occur in any seller-offer period, for exactly the same reason. During buyer-offer periods, however, the informed party has a much richer language with which to communicate, so a much richer class of outcomes becomes possible. There is now a potential for the buyer to signal his type, with higher valuation buyer types trading off higher prices for a higher probability of acceptance. But as in the literature on labor market signaling, many other types of outcomes can be sustained in sequential equilibrium, with different buyer types pooling or partially pooling on common equilibrium offers.

Researchers have long considered many of these equilibria to be implausible, because they are sustained by the threat of adverse inferences following out-of-equilibrium offers. Unfortunately, the literature on refinements has concentrated mostly on pure signaling games (Cho and Kreps, 1987), so there exist few selection criteria applicable to the more complicated extensive-form games we are considering here. In narrowing down the range of equilibrium predictions, researchers have therefore resorted to criteria which try to preserve the spirit of refinements developed for signaling games, but the necessarily ad-hoc nature of those criteria has led to a variety of equilibrium predictions

(Rubinstein, 1985a; Cho, 1990b; Bikchandani, 1992).[12]

To select plausible equilibria, Ausubel and Deneckere (1998) propose a refinement of perfect equilibrium, termed *assuredly perfect equilibrium* (APE). Assuredly perfect equilibrium requires stronger player types (e.g., lower valuation buyer types) to be infinitely more likely to tremble than weaker player types, as the tremble probabilities converge to zero. The purpose of making the strong player types much more likely to tremble is to rule out adverse inferences: following an unexpected move by the uninformed party, beliefs must be concentrated on the strong type, unless this action yields the weak type its equilibrium utility.[13] Thus beliefs are not permitted to shift to the weak type unless there is a reason why (in the equilibrium) the weak type may wish to select the deviant action. APE has the advantage of being relatively easy to apply, and is guaranteed to always exist in finite games.

Importantly, for the two-type alternating-offer bargaining model given by (4), Ausubel and Deneckere (1998) show that for generic priors there exists a unique APE.[14] We will describe this equilibrium outcome here only for the game in which the seller moves first (this facilitates comparison with the seller-offer game). For this purpose, let us define $\tilde{n} = \max\{n \in Z_+ : 1 - \delta^{2n-2} - \delta^{2n-1}\alpha^{-1} < 0\}$. The meaning of $\tilde{n}$ is that in equilibrium, regardless of the fraction of low valuation buyer types, the game always concludes in at most $2\tilde{n}+2$ periods. This should be contrasted with the seller-offer game, where the number of bargaining rounds grows without bound as the seller becomes more and more optimistic.

The intuition behind this difference is that as the number of remaining bargaining rounds becomes larger, the seller extracts more and more surplus from the weak buyer type.[15] At the same time,

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[12] One notable exception is Grossman and Perry (1986a), who develop a general selection criterion, termed perfect sequential equilibrium, and apply it to the alternating-offer bargaining game (1986b). However, perfect sequential equilibria do not generally exist, and in fact fail to do so in the alternating-offer bargaining game when the discount factor is sufficiently high. This is unfortunate, as the case where bargaining frictions become small is of special importance in light of the literature on the Coase Conjecture. General existence is also a problem in Cho (1990b) and Bikchandani (1992).

[13] If an action yields the weak type less than its equilibrium utility, then in approximating games, the weak type must be using that action with minimum probability. As the ratio of the weak to the strong type's tremble probability converges to zero, limiting beliefs will have to be concentrated on the strong type.

[14] More precisely, they show that finite horizon versions of the alternating-offer bargaining game in which the buyer makes the last offer has a unique APE for generic values of the prior. Below, we describe the limit of this equilibrium as the horizon length is approaches infinity.

[15] Formally, this is reflected in the fact that both sequences of prices (6) and (8) are increasing in n, and converge to $\bar{b}$ as n converges to infinity.

there is an upper bound on how much the seller can extract, namely what he would obtain in the complete-information game against the weak buyer type. In the seller-offer game, this is all of the surplus, explaining why with this offer structure the number of effective bargaining rounds can increase without bound as the seller becomes more and more optimistic. In contrast, in the complete-information alternating-offer game the seller receives only a fraction $1/(1+\delta)$ of the surplus (when it is his turn to move). Consequently, in the alternating-offer game the number of effective bargaining rounds must be bounded above, no matter how optimistic the seller.[16] For the sake of brevity, we will consider here only the case where $\tilde{n} > 1$ (note that this necessarily holds when δ is sufficiently high).

Qualitatively, the equilibrium has the following structure. Whenever it is the buyer's turn to make a proposal, all buyer types pool by making nonserious offers, until the seller becomes convinced he is facing the low valuation buyer. At this point, both buyer types pool by making the low valuation buyer's complete-information Rubinstein offer, $r_0 = \delta\, \underline{b}/(1+\delta)$, which the seller accepts. The sequence of prices offered by the seller along the equilibrium path must keep the high valuation buyer indifferent, so we must have:

$$(7) \qquad p_n = (1-\delta^{2n-1})\bar{b} + \delta^{2n-1}\, r_0, \quad n = 1,...,\tilde{n}\ ,$$

unless the seller is extremely optimistic, in which case the game starts out with $\bar{p} = \bar{b}/(1+\delta)$, the seller's offer in the complete-information game against the weak buyer type.

Analogous to the seller-offer game, the sequence of cutoff levels q_n is constructed so that at q_n ($n = 1,...,\tilde{n}$) the seller is indifferent between charging p_n and p_{n-1}, and at $q_{\tilde{n}+1}$ the seller is indifferent between charging $\bar{p}$ and $p_{\tilde{n}}$. Formally, let $q_{-1} = 1$, $q_0 = \hat{q}$, and inductively define the sequence of cutoff levels $q_1 > q_2 > ... > q_{\tilde{n}} > q_{\tilde{n}+1}$ from $m_1 = (\alpha-1)m_0$, $m_2 = \beta\delta^{-1}(1+\delta)^{-1}m_1$,

$$(8) \qquad m_n = \beta\delta^{-(2n-3)}m_{n-1}, \qquad \text{for } 3 \le n \le \tilde{n}\ ,$$

and $m_{\tilde{n}+1} = \omega\, m_{\tilde{n}}$, where $\beta = \bar{b}/[\bar{b}-r_0]$ and $\omega = (1-\delta^2)\bar{b}/[\bar{p}-p_{\tilde{n}}]$. To rule out nongeneric cases, and again analogously to the seller-offer game, let $N = \max\ \{n \le \tilde{n}+1 : q_n \ge 0\}$, and suppose $q_N > 0$:

THEOREM 7 (Ausubel and Deneckere, 1998): Consider the alternating-offer game, and suppose that $q_N > 0$. Then in the unique APE outcome, following histories with no prior observable buyer

[16] Formally, $\tilde{n}$ is the largest integer such that p_n remains below $\bar{p} = \bar{b}/(1+\delta)$, the complete information seller offer against the weak buyer type.

deviations, the buyer uses a stationary acceptance strategy. If $N \leq \tilde{n}$, this acceptance strategy is given by:

$$(9) \qquad P(q) = p_n \qquad q \in (q_n, q_{n-1}],\ 0 \leq n \leq N\ ,$$

$$P(q) = \min\{p_n, \bar{p}\} \qquad q \in [0, q_N]\ .$$

In equilibrium, the seller successively makes the offers $p_N, p_{N-1}, ..., p_1$, with the buyer accepting according to (9) and making nonserious counteroffers until p_1 has been rejected. The buyer then counteroffers r_0, which the seller accepts with probability one.

If $N = \tilde{n}+1$, the buyer's acceptance strategy is given by:

$$(10) \qquad P(q) = p_n \qquad q \in (q_n, q_{n-1}],\ 0 \leq n \leq \tilde{n}\ ,$$

$$P(q) = \bar{p} \qquad q \in [0, q_{\tilde{n}}]\ .$$

In equilibrium, the seller starts out by offering $\bar{p}$, which all buyer types in $[0, q_{\tilde{n}}]$ accept, and all other types reject. Following a nonserious buyer offer, the seller then randomizes between the offers $p_{\tilde{n}}$ and $p_{\tilde{n}-1}$ so as to make the weak buyer type indifferent between accepting and rejecting the previous seller offer.[17] Following the offer $p_{\tilde{n}}$ the seller continues with the offers $p_{\tilde{n}-1}, p_{\tilde{n}-2}, ..., p_1$, and following the offer $p_{\tilde{n}-1}$ the seller continues with the offers $p_{\tilde{n}-2}, ..., p_1$. In each case, the buyer accepts according to (10), and makes nonserious counteroffers until p_1 has been rejected. The game then ends with the buyer counteroffering r_0, which the seller accepts with probability one.

One of the main thrusts of the literature on static signaling models has been to show that refinements based on stability (Kohlberg and Mertens, 1986) tend to select signaling equilibria (Cho and Sobel, 1990). For example, Cho and Kreps (1987) show that in the Spence labor market signaling game with two types, the Intuitive Criterion selects the Pareto efficient separating equilibrium (it is easily verified that APE would select the same outcome). In contrast, in the alternating-offer bargaining game considered above, the buyer uses only fully-pooling offers along the equilibrium path.

The intuition for why pooling obtains is that the strong buyer type *tries* to separate by making a nonserious offer and delaying trade. The only alternative for the weak buyer type is therefore to make a separating offer, which yields the worst possible (complete-information) utility level. Meanwhile,

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[17] In other words, denoting the weight on $p_{\tilde{n}}$ by ϕ, we have $\bar{b} - \bar{p} = \delta^2 \{\phi\ (\bar{b} - p_{\tilde{n}}) + (1-\phi)\ (\bar{b} - p_{\tilde{n}-1})\}$.

stationarity of the equilibrium acceptance strategy provides the seller with an incentive to accelerate trade, and therefore (by the usual Coase Conjecture argument) to charge a relatively low price following rejection of the nonserious offer. But then a revealing offer cannot be optimal, so the equilibrium has to be pooling (see the discussion surrounding Theorem 12 for related intuition).

In fact, from Theorem 7, we can see that the strong version of the Coase Conjecture also holds in the alternating-offer game: there exist a uniform bound M such that regardless of the discount factor δ trade occurs in at most $2M-1$ periods. Indeed, m_n is decreasing in δ for all $n \leq \tilde{n}$, and $\tilde{n}$ converges to infinity as δ converges to 1, so we can find M by recursing m_n at $\delta = 1$. Note that M must be finite, because $m_2(1) = 2\theta m_1$ and $m_n(1) = \theta m_{n-1}(1)$ where $\theta = (1+\alpha^{-1}) > 1$.

It is interesting to compare the effect of shifting bargaining power to the informed party on equilibrium bargaining delay. For this purpose, let us denote the solution to (5) by m_n^s and the solution to (8) by m_n^a. Observe that $m_0^s = m_0^a$ and $m_1^s = m_1^a$; some straightforward but tedious algebra shows that $m_n^s(\delta) < m_n^a(\delta)$, for $n \geq 2$. We conclude that as long as the alternating-offer game starts out with a seller offer below $\overline{p}$,[18] the alternating-offer game requires more offers, and has a lower acceptance probability than the seller-offer game. Moreover, the alternating-offer game results in additional delay because (with the exception of the final bargaining round) only seller-offer periods result in trade. Hence the traditional wisdom that bargaining becomes more efficient as the informed party gains bargaining strength proves to be incorrect.

Finally, it should be noted that when the seller is so optimistic that she starts with the highest possible offer $\overline{p}$, the equilibrium requires her to randomize with positive probability two periods later. Unlike in the seller-offer game, randomization in seller offers may thus be necessary along the equilibrium path.

3.1.3 The Buyer-Offer Game and Other Extensive Forms

In the game where the buyer makes all the offers, it is clearly a sequential equilibrium for the buyer to always offer the seller his cost c, and for the seller to accept any price above c with probability one. Ausubel and Deneckere (1989b, Theorem 4) show that this is in fact the only sequential equilibrium. Intuitively, the seller can do no better than in the complete-information game where the buyer is known to have valuation $v(1)$, but since the buyer makes all the offers, he can

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[18] As discussed above, this is necessarily the case when δ is sufficiently large.

extract all of the surplus no matter what his valuation. We conclude that the buyer-offer game always achieves an efficient outcome, regardless of whether or not there is a gap, condition (L) holds, or the magnitude of the discount factor.

More generally, we can study the impact of transferring bargaining power from the seller to the buyer by considering the (k,l)-alternating-offer bargaining game, in which the seller and buyer alternate by making k and l successive offers, respectively. The ratio k/l measures the relative frequency with which the seller gets to make offers, and hence is a measure of his bargaining strength. Note that in the complete-information case, this game yields the same outcome as the alternating-offer game in which the seller's discount factor is given by $\delta_S = \delta^l$ and the buyer's discount factor is given by $\delta_B = \delta^k$. Thus, the ratio $\rho \equiv k/l$ can also be interpreted as the relative degree of impatience between the bargaining parties. Observe now that in any sequential equilibrium, the weakest buyer type must earn at least what he would in the complete-information case, so we have $U(1) \geq v(1)\ \delta_S(1-\delta_B)/(1-\delta_S\delta_B)$, which converges to $v(1)/(1+\rho)$ as δ approaches 1. Since $v(1)$ is the maximum surplus available, we conclude that when ρ is small all sequential equilibria must yield bargaining outcomes that are nearly efficient. This conclusion obtains regardless of whether or not there is a gap.

Admati and Perry (1987) consider an alternating-offer extensive-form game that differs from Rubinstein's game in that the length between successive offers is chosen endogenously by the players. Thus, when a player rejects an offer, he commits unilaterally to neither make a counteroffer nor receive another offer until a length of time of his choice has elapsed. During this time period, all communications are closed off, and the commitment is irrevocable. Admati and Perry analyze the two-type model given by (4), and apply a forward-induction-like refinement. When the prior on the weak type is sufficiently high, this refinement uniquely selects a separating equilibrium.[19] The seller starts out by making the offer $\overline{p} = \overline{b}/(1+\delta)$, which the weak buyer type accepts, and the strong buyer type rejects. The strong buyer type then delays any further negotiation until a time of length T has elapsed, at which point it makes its complete-information counteroffer $r_0 = \delta\ \underline{b}/(1+\delta)$, which the seller accepts. T is chosen such that the weak buyer type is indifferent between accepting the seller's initial offer, and mimicking the low buyer type. This equilibrium has an intuitive structure strongly reminiscent of the Riley outcome in the Spence labor market signaling model, but this elegance comes at a strong price: the buyer is committed not to receive any counteroffer during the time interval of length T. Note that the seller has an incentive to make such a counteroffer, for once the buyer has

[19] For intermediate values of the prior, there are multiple equilibria, and for sufficiently low values of the prior the seller offers the strong buyer's complete-information price.

chosen T, his type is revealed to be strong. In fact, both parties would be better off settling immediately at the price r_0, and the buyer knows this is the case, but is committed not to reopen the lines of communication until time T (Admati and Perry, 1987, Section 8.4). If the communication channels were allowed to reopen any earlier, the signaling equilibrium would be destroyed. Indeed, in the alternating-offer game analyzed in the previous section, separation never occurs.

3.2 Interdependent Values

Consider the trading situation in which a seller who is privately informed about the quality of a used car faces a potential buyer who cares about the quality of the vehicle. As we saw in Section 2, there then exists a trading mechanism that can achieve the efficient outcome if and only if the buyer's expected valuation exceeds the valuation of the owner of the highest quality car, i.e., $E[v(q)] \geq c(1)$. This raises two interesting questions for extensive-form bargaining. First, assuming that the above condition holds, will the same forces that operate in the private values model to produce efficient trade when bargaining frictions disappear still permit the efficient outcome to be reached when values are interdependent? Second, assuming that the above condition is violated, will the limiting trading outcome at least be ex ante efficient, in the sense that it maximizes the expected gains from trade subject to the IC and IR constraints?

So far, the literature has only studied the bargaining game in which the uninformed party (the buyer) makes all the offers (Evans, 1989; Vincent, 1989). Our discussion here is based upon Deneckere and Liang (1999). The arguments establishing existence of equilibrium for the interdependent values with a gap closely parallel those of Gul, Sonnenschein and Wilson (1986). Consequently an analogue of Theorem 6 holds, with $c(q)$ taking the role of $v(q)$, with one important difference: it is no longer the case that the number of bargaining rounds is uniformly bounded above, regardless of the discount factor (if this were the case, then as the discount factor converged to one, the efficient outcome would obtain even when $E[v(q)] < c(1)$, contradicting Theorem 2). As in the private values case, generically there is a unique equilibrium outcome, and equilibrium outcomes are sustained by a unique stationary triplet. In equilibrium, the buyer successively increases his offers over time. Low-quality seller types accept low prices, while high-quality seller types suffer delay in order to credibly prove they possess a higher-quality vehicle. The intuition for uniqueness is analogous to the one given in Section 3.1.1: under condition (L), once the buyer's beliefs cross a threshold level, he finds it no longer worthwhile to price discriminate among the remaining seller types.[20]

To illustrate consider the simple two-type model:

$$c(q) = 0 \qquad v(q) = \alpha \qquad , \text{ for } 0 \le q \le \hat{q} ,$$

$$c(q) = s \qquad v(q) = s + \beta \quad , \text{ for } \hat{q} < q \le 1 ,$$

where $\alpha > 0$ and $\beta > 0$, since we are in the case of a gap. Note that the private values case obtains when $\alpha = s + \beta$, so this is a generalization of the example studied in Section 3.1.1. See Evans (1989) for a treatment of the special case in which $\alpha = 0$.

Let $q_{-1} = 1$, $q_0 = \hat{q}$, and inductively define the sequence $q_1 > q_2 > ... > q_N$ from:

$$m_n = \alpha s^{-1} \delta^{-(n-1)} m_{n-1} \qquad (n \ge 2),$$

and the initial condition $m_1 = \beta s^{-1} m_0$, using $m_n = q_{n-1} - q_n$ and the terminal condition $N = \min\{n : q_n \le 0\}$. Finally, let $p_n = s\ \delta^n$. Then with every sequential equilibrium is associated the unique stationary triplet:

$$P(Q) = p_n \qquad Q \in (q_n, q_{n-1}] ;$$

$$t(Q) = q_{n-2} \qquad Q \in (q_n, q_{n-1}] \text{ and } n > 1 ,$$

$$= 1 \qquad Q \in (q_1, 1] ;$$

$$V(Q) = (\alpha - p_{n-1})(q_{n-2} - Q) + \delta\ V(q_{n-2}) \qquad Q \in (q_n, q_{n-1}] \text{ and } n > 1 ,$$

$$= (\alpha - p_0)(q_0 - Q) + \beta\ (1 - q_0) \qquad Q \in (q_1, q_0] ,$$

$$= \beta\ (1 - Q) \qquad Q \in (q_0, 1] .$$

The idea behind the above construction is as follows. Seller types in $(\hat{q},1]$ are held to their reservation value, because the buyer has the sole power to make offers. The last price offered will therefore be equal to s. Seller types in the interval $[0,\hat{q}]$ must be indifferent between accepting the offer p_n and waiting n periods to receive the offer $p_0 = s$, so we must have $p_n = s\ \delta^n$. The breakpoints q_n are constructed so that when the state is q_n the buyer is indifferent between offering p_n (and hence trading with types in $(q_n, q_{n-1}]$), and offering p_{n-1} (and hence trading with types in $(q_{n-1}, q_{n-2}]$.

[20] See Samuelson (1984) for a generalization of Stokey's "no price discrimination" result to the interdependent values case.

Note that, in the private-values case, the sequence $\{m_1, m_2, ...\}$ is strictly increasing and bounded below when δ converges to 1. This is still the case here when $\alpha \geq s$. But when $\alpha < s$, the sequence is decreasing as long as n remains such that $\delta^n < \alpha/s$. As δ converges to 1, the range of integers for which this inequality holds increases without bound, so it is possible for the number of bargaining rounds to increase without bound as δ converges to 1. This allows us to investigate the conditions under which the Coase Conjecture will and will not hold. For this purpose, let us explicitly denote the dependence of m_i on δ by $m_i(\delta)$, and define:

$$a = \sum_{i=0}^{\infty} m_i(1) = (1 - \hat{q})(1 + \frac{\beta}{s - \alpha}) .$$

We then have:

THEOREM 8 (Deneckere and Liang, 1999): Consider the two-type interdependent values model defined above. Then the Coase Conjecture obtains if and only if $a \geq 1$. When $a < 1$, then as δ converges to 1, all seller types in $[0,1-a)$ trade immediately at the price $s\rho^2$, and all types in $(1-a,1]$ trade at the price s after a delay of length T discounted such that $e^{-rT} = \rho^2$, where $\rho = \alpha/s$.

The condition $a \geq 1$ can be written in the more familiar form $E(v(q)) \geq c(1)$, so Theorem 8 says that when bargaining frictions disappear, inefficient delay occurs if and only if this is mandated by the basic incentive constraints presented in Theorem 2. When $E(v(q)) < c(1)$ every trading mechanism necessarily exhibits inefficiencies. However, the limiting bargaining mechanism described in Theorem 8 exhibits more delay than is necessary. To see this, observe that social welfare is increased by having all types $q \in (1-a, \hat{q}]$ trade at the price $s\rho^2$ at time zero. In the resulting mechanism the buyer will have strictly positive surplus; this means we can increase the probability of trade on the interval $(\hat{q},1]$ and thereby further increase welfare.

4 Sequential Bargaining with One-Sided Incomplete Information: The "No Gap" Case

The case of *no gap* between the seller's valuation and the support of the buyer's valuation differs in broad qualitative fashion from the case of the *gap* which we examined in the previous section. The bargaining does not conclude with probability one after any finite number of periods. As a consequence of this fact, it is not possible to perform backward induction from a final period of trade, and it therefore does not follow that every sequential equilibrium need be stationary. If stationarity is nevertheless *assumed*, then the results parallel the results which we have already seen for the *gap* case: trade occurs with essentially no delay and the informed party receives essentially all the surplus. However, if stationarity is not assumed, then instead a folk theorem obtains, and so substantial delay in trade is possible and the uninformed party may receive a substantial share of the surplus. These qualitative conclusions hold both for the seller-offer game and alternating-offer games.

Following the same convenient notation as in Section 3, let the buyer's type be denoted by q, which is uniformly distributed on [0,1], and let the valuation of buyer type q be given by the function $v(q)$. The seller's valuation is normalized to equal zero. The case of "no gap" is the situation where there does *not* exist $\Delta > 0$ such that it is common knowledge that the gains from trade are at least Δ. More precisely, for any $\Delta > 0$, there exists $q_\Delta \in [0,1)$ such that $0 < v(q_\Delta) < \Delta$. Opposite the conclusion of Theorem 4 for the gap case, we have:

LEMMA 2: In any sequential equilibrium of the infinite-horizon seller-offer game in the case of "no gap," and for any $N < \infty$, the probability of trade before period N is strictly less than one.

PROOF: By Lemma 1, at the start of any period t, the set of remaining buyer types is an interval $(Q_t,1]$. The seller never offers a negative price (Fudenberg, Levine and Tirole, 1985, Lemma 1). Consequently, a price of $(1 - \delta)v(q) - \varepsilon$ will be accepted by all buyer types less than q, since a buyer with valuation $v(q)$ is indifferent between trading at a price of $(1 - \delta)v(q)$ in a given period and trading at a price of zero in the next period.

Suppose, contrary to the Lemma, that there exists finite integer N such that $Q_N = 1$. Without loss of generality, let N be the smallest such integer, so that $Q_{N-1} < 1$. Since acceptance is individually rational, the seller must have offered a price of zero in period $N-1$, yielding zero continuation payoff. But this was not optimal, as the seller could instead have offered $(1 - \delta)v(q) - \varepsilon$ for some $q \in (q_{N-1},1)$, generating a continuation payoff of at least $(q - Q_{N-1})[(1 - \delta)v(q) - \varepsilon] > 0$ (for sufficiently small ε), a contradiction. We conclude that $Q_N < 1$. ■

A result analogous to Lemma 2 also holds in the alternating-offer extensive-form. However, as we have already seen in Section 3.1.3, the result for the buyer-offer game is qualitatively different: there is a unique sequential equilibrium; it has the buyer offering a price of zero in the initial period and the seller accepting with probability one.

Much of the intuition for the case of "no gap" can be developed from the example where the seller's valuation is commonly known to equal zero and the buyer's valuation is uniformly distributed on the unit interval [0,1]. This example was first studied by Stokey (1981) and Sobel and Takahashi (1983). In our previous notation:

(11) $$v(q) = 1 - q, \text{ for } q \in [0,1].$$

In the subsections to follow, we will see that the stationary equilibria are qualitatively similar to those for the "gap" case, but that the nonstationary equilibria may exhibit entirely different properties.

4.1 Stationary Equilibria

Assuming a stationary equilibrium and given the linear specification of Eq. (11), it is plausible to posit that the seller's value function ($V(Q)$) is quadratic in the measure of remaining customers, that the measure of remaining customers ($1 - t(Q)$) which the seller chooses to induce is a constant fraction of the measure of currently-remaining customers, and that the seller's optimal price ($P(t(Q))$) is linear in the measure of remaining customers. Let r denote the real interest rate and z denote the time interval between periods (so that the discount factor δ is given by $\delta \equiv e^{-rz}$). In the notation of Section 3:

(12) $$V(Q) = \alpha_z(1 - Q)^2,$$

(13) $$1 - t(Q) = \beta_z(1 - Q),$$

(14) $$P(t(Q)) = \gamma_z(1 - Q),$$

where α_z, β_z and γ_z are constants between 0 and 1 which are parameterized by the time interval z between offers. Eqs. (12)-(13)-(14) can be solved simultaneously, as follows. Since the linear-quadratic solution is differentiable and $t(Q)$ is defined to be the arg max of Eq. (2), we have:

(15) $$\frac{\partial}{\partial Q'}\left[P(Q')\cdot(Q' - Q) + \delta V(Q')\right]_{Q' = t(Q)} = 0\,.$$

Furthermore, with $t(Q)$ substituted into the right-hand-side of Eq. (2), the maximum must be attained:

(16) $$V(Q) = P(t(Q))\cdot(t(Q) - Q) + \delta V(t(Q))\,.$$

Substituting Eqs. (12), (13) and (14) into Eqs. (3), (15) and (16) yields three simultaneous equations in

α_z, β_z and γ_z, which have a unique solution. In particular, the solution has $\alpha_z = ½\,\gamma_z$ and:

(17) $$\gamma_z = 1 - \delta^{-1} + \delta^{-1}\sqrt{1-\delta}\;.$$

(Stokey, 1981, Theorem 4; and Gul, Sonnenschein and Wilson, 1986, pp. 163−64).

Qualitatively, the reader should observe that in the limit as the time interval z between offers approaches zero (i.e., as $\delta \to 1$), γ_z converges to zero. From Eq. (14), observe that γ_z is the seller's price when the state is $Q = 0$. This means that the *initial* price in this equilibrium may be made arbitrarily close to zero (i.e., the Coase Conjecture holds). Moreover, since $\alpha_z = ½\,\gamma_z$, the seller's expected profits in this equilibrium may be made arbitrarily close to zero. According to (17), the convergence is relatively slow, but for realistic parameter values, the seller loses most of her bargaining power. For example, with a real interest rate of 10% per year and weekly offers, the seller's initial price is 4.2% of the highest buyer valuation; this diminishes to 1.63% with daily offers.

Further observe that, since the linear-quadratic equilibrium is expressed as a triplet $\{P(\bullet),V(\bullet),t(\bullet)\}$, this sequential equilibrium is stationary. However, this model is also known to have a continuum of other stationary equilibria; see Gul, Sonnenschein and Wilson (1986, Examples 2 and 3). Unlike the other known stationary sequential equilibria, the linear-quadratic equilibrium has the property that it does not require randomization off the equilibrium path. In the literature, stationary sequential equilibria possessing this arguably-desirable property are referred to as *strong-Markov* equilibria; while stationary sequential equilibria not necessarily possessing this property are often referred to as *weak-Markov* equilibria.

The linear-quadratic equilibrium of the linear example is emblematic of all stationary sequential equilibria for the case of "no gap," as the following theorem shows:

THEOREM 9 (Coase Conjecture): For every $v(\bullet)$ in the case of "no gap" and for every $\varepsilon > 0$, there exists $\overline{z} > 0$ such that, for every time interval $z \in (0,\overline{z})$ between offers and for every stationary sequential equilibrium, the initial price charged in the seller-offer game is less than ε.

PROOF: Gul, Sonnenschein and Wilson (1986), Theorem 3.

If extremely mild additional assumptions are placed on the valuation function of buyer types, then a stronger version of the Coase Conjecture can be proven. The standard Coase Conjecture may be viewed as establishing an upper bound on the ratio between the seller's offer and the highest buyer valuation in the *initial period*; the *uniform* Coase Conjecture further bounds the ratio between the seller's offer and the highest-remaining buyer valuation in *all periods* of the game. For L, M and α such that $0 < M \leq 1 \leq L < \infty$ and $0 < \alpha < \infty$, let:

$$(18) \qquad \mathscr{F}_{L,M,\alpha} = \{\ v(\bullet) : v(0) = 1,\ v(1) = 0 \text{ and } M(1-q)^{\alpha} \leq v(q) \leq L(1-q)^{\alpha} \text{ for all } q \in [0,1]\}.$$

The family $\mathscr{F}_{L,M,\alpha}$ has the property that if $v \in \mathscr{F}_{L,M,\alpha}$, then *every* truncation (from above) of the probability distribution of buyer valuations (renormalized so that the valuation at the truncation point equals one) is guaranteed to also be an element of $\mathscr{F}_{L/M,M/L,\alpha}$. If a uniform $\bar{z}$ (of Theorem 9) can be found which holds for all $v \in \mathscr{F}_{L/M,M/L,\alpha}$, then the ratio between the seller's offer and the highest-remaining buyer valuation is bounded by ε in all periods of the game. We have:

THEOREM 10 (Uniform Coase Conjecture): For every $0 < M \leq 1 \leq L < \infty$, $0 < \alpha < \infty$, and $\varepsilon > 0$, there exists $\bar{z} > 0$ such that for every time interval $z \in (0,\bar{z})$ between offers, for every $v \in \mathscr{F}_{L,M,\alpha}$ and for every stationary sequential equilibrium, the initial price charged in the seller-offer game is less than ε.

PROOF: Ausubel and Deneckere (1989a), Theorem 5.4.

The same qualitative results hold in alternating-offer extensive forms for the case of no gap. Some additional assumptions above and beyond stationarity are made in the literature, but the stationarity assumption appears to be the driving force behind the results. Gul and Sonnenschein (1988), in analyzing the gap case, and Ausubel and Deneckere (1992a) assume stationarity.[21] They also assume that the seller's offer and acceptance rules are in pure strategies,[22] and that there is "no free screening" in the sense that any two buyer offers which each have zero probability of acceptance are required to induce the same beliefs. Similar to the seller-offer game, these imply:

[21] To be more precise, they assume requirement (3) and a slightly weaker version of requirement (2) in the definition of stationarity from Section 3. Their assumptions of pure strategies and no free screening imply requirement (1).

[22] In light of Section 3.1.2, the pure strategy assumption on seller acceptances may be inconsistent with refinements of sequential equilibrium; but a similar result likely holds under weaker assumptions.

THEOREM 11 (Uniform Coase Conjecture): For every $0 < M \leq 1 \leq L < \infty$, $0 < \alpha < \infty$, and $\varepsilon > 0$, there exists $\bar{z} > 0$ such that for every time interval $z \in (0,\bar{z})$ between offers, for every $v \in \mathscr{F}_{L,M,\alpha}$ and for every stationary sequential equilibrium, the initial serious (seller or buyer) offer in the alternating-offer game is less than ε.

PROOF: Ausubel and Deneckere (1992a), Theorem 3.2.

For the no gap case, the Coase Conjecture is equivalent to the notion of "No Delay" which Gul and Sonnenschein (1988) prove for the gap case: for sufficiently short time interval between offers, the probability that trade will occur within ε time is at least $1 - \varepsilon$. This equivalence holds in the seller-offer as well as in the alternating-offer game.

There is an especially enlightening explanation for the fact that stationary sequential equilibria of the alternating-offer game closely resemble those of the seller-offer game. In a sense which may be made precise, stationary equilibria of the alternating-offer game are *as if* the extensive form permitted offers only by the uninformed party: exogenously, both traders are permitted to make offers; endogenously, equilibrium counteroffers by the informed party degenerate to null moves.

To see this, observe that the stationarity, pure-strategy and no-free-screening restrictions on sequential equilibrium mandate that, at each time when it is the informed agent's turn to make an offer, the informed agent partitions the interval of remaining types into two subintervals (one possibly degenerate): a high subinterval (who "speak" by making a serious offer) and a low subinterval (who effectively "remain silent" by making a nonserious offer). Choosing to speak reveals a high valuation, which is information that the uninformed agent can exploit in the ensuing negotiations. Remaining silent signals a low valuation. Let $\underline{b}$ denote the lowest buyer valuation in the speaking subinterval—as well as the highest buyer valuation in the silence subinterval. Following speaking, the seller captures a price of at least $\underline{b}\delta/(1+\delta)$, à la Rubinstein (1982), which as the time between offers shrinks toward zero, converges to $\frac{1}{2}\underline{b}$. Meanwhile, also as the time between offers shrinks toward zero, the terms of trade for the silence interval become increasingly favorable: à la the Uniform Coase Conjecture, the ratio between the next price and $\underline{b}$ converges to zero. Thus, silence becomes increasingly attractive relative to speaking and, for sufficiently short time intervals, delay becomes preferable to revealing the damaging information for *all* types of the informed party. In other words, you recognize that "anything you say can and will be used against you." Therefore, regardless of valuation, you decline to speak, since "you have the right to remain silent." More formally:

THEOREM 12 (Silence Theorem): Let v belong to $\mathscr{F}_{L,M,\alpha}$ and let r be any positive interest rate. Then there exists $\bar{z} > 0$ such that, for every time interval $z \in (0,\bar{z})$ between offers and for every stationary sequential equilibrium satisfying the pure-strategy and no-free-screening restrictions, the informed party never makes any serious offers in the alternating-offer bargaining game, both along the equilibrium path and after all histories in which no prior buyer deviations have occurred.

PROOF: Ausubel and Deneckere (1992a), Theorem 3.3.

Thus, stationary equilibria of the alternating-offer bargaining game with a time interval z between offers closely resemble stationary equilibria of the seller-offer bargaining game with a time interval $2z$ between offers, for sufficiently small z. Moreover, for many distributions of valuations, "sufficiently" small does not require "especially" small: for the model with linear $v(\bullet)$, the silence theorem holds whenever $\delta > 0.83929$ (Ausubel and Deneckere, 1992a, Table I); with a real interest rate r of 10% per year, this holds for all $z < 21$ months, not requiring a very quick response time between offers at all.

4.2 Nonstationary Equilibria

In the case of no gap, stationarity is merely an assumption, not an implication of sequential equilibrium. As we saw in the last subsection, the stationary equilibria converge (as the time interval between offers approaches zero) in outcome to the static mechanism which maximizes the *informed* party's expected surplus. The contrast between stationary and nonstationary equilibria is most sharply highlighted by constructing nonstationary equilibria which converge in outcome to the static mechanism which maximizes the *uninformed* party's expected surplus.

Again, consider the example where the seller's valuation is commonly known to equal zero and the buyer's valuation is uniformly distributed on the unit interval [0,1]. The static mechanism which maximizes the seller's expected surplus is given by:

(19) $$p(q) = 1 \text{ , if } q \le \tfrac{1}{2} \text{ ,} \qquad x(q) = \tfrac{1}{2} \text{ , if } q \le \tfrac{1}{2} \text{ ,}$$
$$= 0 \text{ , if } q > \tfrac{1}{2} \text{ ,} \qquad = 0 \text{ , if } q > \tfrac{1}{2} \text{ .}$$

In terms of a sequential bargaining game, this means that, although it is possible to intertemporally price discriminate, the seller finds it optimal to merely select the static monopoly price of ½ and adhere to it forever (Stokey, 1979). The intuition for this result—in terms of the durable goods monopoly interpretation of the model—is that the sales price for a durable good equals the discounted sum of the period-by-period rental prices, and the optimal rental price for the seller in each period is always the same monopoly rental price.

A seller who lacks commitment powers will be unable to follow precisely this price path (Coase, 1972). If the seller were believed to be charging prices of $p_n = ½$, for $n = 0,1,2, \ldots$, the unique optimal buyer response would be for all $q \in [0,½)$ to purchase in period 0 and for all $q \in (½,1]$ to never purchase (corresponding exactly to the static mechanism of Eq. (19)). But, then, the seller's continuation payoff evaluated in any period $n = 1,2,3, \ldots$ equals zero literally. Following the same logic as in the proof of Lemma 2, there exists a deviation which yields the seller a strictly positive payoff, establishing that the constant price path is inconsistent with sequential equilibrium.

However, while the static mechanism of Eq. (19) cannot literally be implemented in equilibria with constant price paths, Ausubel and Deneckere (1989a) show that the seller's optimum can nevertheless be arbitrarily closely approximated in equilibria with slowly-descending price paths. The key to their construction is as follows. For any $\eta > 0$, and in the game with time interval $z > 0$ between offers, define a *main equilibrium path* by:

(20) $$p_n = p_0\, e^{-\eta n z}, \quad \text{for } n = 0,1,2, \ldots .$$

Also consider the (linear-quadratic) stationary equilibrium which was specified in Eqs. (12)-(13)-(14) and in which γ_z was solved for in Eq. (17). Define a *reputational price strategy* by the following seller strategy:

(21) Offer p_m in period m, if p_n was offered in all periods $n = 0,1, \ldots ,m-1$,

Offer prices according to the stationary equilibrium, otherwise,

with the corresponding buyer strategy defined to optimize against the seller strategy (21).

It is straightforward to see that, for sufficiently short time intervals between offers, the reputational price strategy yields a (nonstationary) sequential equilibrium. This is the case for all $p_0 \in (0,1)$; and for $p_0 = ½$, the sequential equilibrium converges in outcome (as $\eta \to 0$ and $z \to 0$) to the static mechanism (19) which maximizes the seller's expected payoff. A heuristic argument proceeds as follows. First, observe that the price path $\{p_n\}_{n=0}^{\infty}$ yields a relatively large measure of sales in period 0 and then a relatively slow trickle of sales thereafter. Hence, if the main equilibrium path is self-enforcing for the seller in periods $n = 1,2, \ldots$, it will automatically be self-enforcing in period $n = 0$. Second, let us consider the seller's continuation payoff along the main equilibrium path, evaluated in any period $n = 1,2, \ldots$. Let q denote the state at the start of period n. Given the linear distribution of types and the exponential rate of descent in price, it is easy to see that the seller's expected continuation payoff, π, is a stationary function of the state:

(22) $$\pi(q) \; = \; \lambda_z \,(1 - q)^2 \,,$$

where λ_z depends on η and is parameterized by z. Moreover, for every $\eta > 0$:

(23) $$\lambda \; \equiv \; \lim_{z \to 0} \lambda_z \; > \; 0 \,.$$

Meanwhile, we already saw in Eq. (12) that the seller's payoff from optimally deviating from the main equilibrium path is given by $V(q) \; = \; \alpha_z(1 - q)^2$, where $\alpha_z \to 0$ as $z \to 0$. Thus, for any $\eta > 0$, there exists $\bar{z} > 0$ such that, whenever the time interval between offers satisfies $0 < z < \bar{z}$, we have $\lambda_z > \alpha_z$, and so the seller's expected payoff along the main equilibrium path exceeds the expected payoff from optimally deviating. We then conclude that the reputational price strategy yields a sequential equilibrium.

This construction generalizes to all valuation functions $v \in \mathscr{F}_{L,M,\alpha}$ and to all bargaining mechanisms. It is appropriate to restrict attention here to incentive-compatible bargaining mechanisms that are *ex post* individually rational, since the buyer will never accept a price above his valuation in any sequential equilibrium and the seller will never offer a price below her valuation. Continuing the logic developed in Theorem 2,[23] we have the following complete characterization:

LEMMA 3: For any continuous valuation function $v(\bullet)$, the one-dimensional bargaining mechanism $\{p,x\}$ is incentive compatible and *ex post* individually rational if and only if $p : [0,1] \to [0,1]$ is (weakly) decreasing and x is given by the Stieltjes integral: $x(q) \; = \; -\int_q^1 v(r)\, dp(r)$.

PROOF: Ausubel and Deneckere (1989b), Theorem 1.

[23] The analogue to Lemma 3 for the case where the seller is informed and the buyer uninformed follows directly from Theorem 2, as follows. With private values, i.e., $g(s) = b$ for all s, the first inequality in Theorem 2 is automatically satisfied. Since we are in the case of no gap, seller type $\bar{s}$ cannot profitably trade with the buyer, so ex post individual rationality requires $x(\bar{s}) - \bar{s}\,p(\bar{s}) = 0$. Consequently, it follows from Theorem 2 that:

$$x(s) \; = \; s \; p(s) \; + \int_s^{\bar{s}} p(z)\, dz \; = \; \bar{s} \; p(\bar{s}) \; - \int_s^{\bar{s}} z\, dp(z) \;.$$

Moreover, we can translate the outcome path of any sequential equilibrium of the bargaining game into an incentive-compatible bargaining mechanism, as follows. For buyer type q, let $n(q)$ denote the period of trade for type q in the sequential equilibrium and let $\phi(q)$ denote the payment by type q. Define: $\overline{p}(q) = e^{-r\, n(q)z}$ and $\overline{x}(q) = \phi(q)\, e^{-r\, n(q)z}$. Then $\{\overline{p},\overline{x}\}$ thus defined can be reinterpreted as a direct mechanism—and the fact that it derives from a sequential equilibrium immediately implies that $\{\overline{p},\overline{x}\}$ is incentive-compatible and individually-rational. We will say that $\{p,x\}$ is *implemented* by sequential equilibria of the bargaining game if, for every $\varepsilon > 0$, there exists a sequential equilibrium inducing static mechanism $\{\overline{p},\overline{x}\}$ with the property that $\{\overline{p},\overline{x}\}$ is uniformly close to $\{p,x\}$ (except possibly in a neighborhood of $q = 1$):

(24) $$|\overline{p}(q) - p(q)| < \varepsilon,\ \forall q \in [0,1-\varepsilon) \text{ and } |\overline{x}(q) - x(q)| < \varepsilon,\ \forall q \in [0,1].$$

The reasoning described above for the static mechanism which maximizes the seller's expected payoff extends to *every* incentive-compatible bargaining mechanism. In place of the exponentially-descending price path $\{p_n\}_{n=0}^{\infty}$, we substitute a general specification which approximates the incentive-compatible bargaining mechanism, for $q \in [0,1-\varepsilon]$ and induces an exponential evolution of the state, for $q \in (1-\varepsilon,1]$. In place of the linear-quadratic equilibrium following deviations, we substitute a stationary equilibrium, which is guaranteed to exist (Theorem 4) and to satisfy the Uniform Coase Conjecture (Theorem 10). We have:

THEOREM 13 (Folk Theorem): Let the valuation function $v(\bullet)$ belong to $\mathscr{F}_{L,M,\alpha}$. Then every incentive-compatible, *ex post* individually-rational bargaining mechanism $\{p,x\}$ is implementable by sequential equilibria of the seller-offer bargaining game.

PROOF: Ausubel and Deneckere (1989b), Theorem 2.

A folk-theorem-like result also holds in the alternating-offer game, since each of a continuum of sequential equilibria from the seller-offer game can be embedded as equilibria in the alternating-offer game. At the same time, there is an upper bound on the price at which the buyer can be expected to trade. Suppose that the seller holds the most "optimistic" beliefs: the buyer's type equals 0 and so the buyer's valuation equals $v(0)$. Then, even in the complete-information game, if the seller offers any price greater than $(1/(1+\delta))v(0)$, the buyer is sure to turn around and reject (Rubinstein, 1982). In the limit as the time interval approaches zero, the seller can extract no more than one-half the surplus from the highest-valuation buyer. Thus, we have:

THEOREM 14: Let the valuation function $v(\bullet)$ belong to $\mathscr{F}_{L,M,\alpha}$. Then an incentive-compatible, *ex post* individually-rational bargaining mechanism $\{p,x\}$ is implementable by sequential equilibria of the alternating-offer bargaining game if and only if: $p(0)v(0) - x(0) \geq \frac{1}{2} v(0)$.

PROOF: Ausubel and Deneckere (1989b), Theorem 3.

4.3 Discussion of the Stationarity Assumption

One useful way to understand the effect of the stationarity assumption is to see its impact on the set of equilibria of a standard supergame. Consider, for example, the infinitely-repeated prisoners' dilemma—or any infinite supergame in which the stage game has a unique Nash equilibrium. Since (unlike the bargaining game) this is literally a repeated game and the play of one period has no effect on the possibilities in the next, there is no state variable at all. Stationarity restricts attention to equilibria in which the play in any period is history-independent; in other words, trigger-strategy equilibria are ruled out by assumption. (Equivalently, as in the bargaining game with one-sided incomplete information, stationarity restricts attention to equilibria of the infinite-horizon game which are limits of equilibria of finite-horizon versions of the same game.) The unique stationary equilibrium is the static Nash equilibrium played over and over.

This analogy strongly suggests that it is wrong to assume away the nonstationary equilibria. While it is interesting to know the implications of stationarity, a restriction to stationarity excludes many of the interesting effects which led economists to analyze dynamic games in the first place. Of course, stationarity is essential to the analysis of the "gap" case, since it is implied (not assumed). But to the extent that "no gap" is the appropriate condition on primitives, nonstationary equilibria and their qualitative properties are an essential part of the analysis.

5 Sequential Bargaining with Two-Sided Incomplete Information

With two-sided incomplete information, incentive compatibility and individual rationality are incompatible with ex post efficiency. As we saw in Corollary 1 of Section 2, so long as the supports of the seller and buyer valuations overlapped, the static bargaining mechanism necessarily entails situations where the buyer's valuation exceeds the seller's valuation yet trade occurs with probability strictly less than one. Furthermore, as we saw in the fourth paragraph following Theorem 2, since any sequential equilibrium of the dynamic bargaining game can be expressed as a static mechanism, this immediately implies that the search for ex post efficient sequential equilibria is fruitless. The more interesting starting-point is to ask: Can the ex ante efficient static bargaining mechanism be replicated in a dynamic offer/counteroffer bargaining game, or does the dynamic game necessarily entail greater inefficiency than the static constrained optimum?

Ausubel and Deneckere (1993a) establish that, for distribution functions exhibiting monotonic hazard rates, the ex ante efficient static bargaining mechanism can essentially be replicated in very simple dynamic bargaining games:

THEOREM 15: If $F_1(s)/f_1(s)$ and $[F_2(b) - 1]/f_2(b)$ are strictly increasing functions, then:

(i) there exists $\lambda_s \in (0,1)$ such that, for every $\lambda \in [\lambda_s,1]$, the ex ante efficient mechanism which places weight λ on the seller is implementable in the seller-offer game; and

(ii) there exists $\lambda_b \in (0,1)$ such that, for every $\lambda \in [0,\lambda_b]$, the ex ante efficient mechanism which places weight λ on the seller is implementable in the buyer-offer game.

PROOF: Ausubel and Deneckere (1993a), Theorem 3.1.

The flavor of this result is most easily seen in the standard example where the seller and buyer valuations are each uniformly distributed on the unit interval. For this special case of the theorem, calculations reveal that $\lambda_s = ½ = \lambda_b$. This means that, for the case of equal weighting ($\lambda = ½$) focused on by Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983), we can come arbitrarily close to replicating the constrained optimum both in the seller-offer game and the buyer-offer game. Moreover, since equilibria of the seller- and buyer-offer games can be embedded in sequential equilibria of the alternating-offer game, this means that the entire ex ante Pareto frontier is implementable in the alternating-offer bargaining game. There need not be any additional inefficiency arising from the dynamic nature of the game, above and beyond the inefficiency already introduced by the two-sided incomplete information.

While the (upper) boundary of the set of all sequential equilibria is thus known, little exists in the way of results refining the set of sequential equilibrium outcomes. Cramton (1984) posited sequential equilibria of the infinite-horizon seller-offer bargaining game with the additional properties that: (a) the seller fully reveals her type in the course of making offers; and (b) in the continuation game following the seller's revelation, players adopt the strategies from a stationary equilibrium of the game of one-sided incomplete information. The seller thus uses delay to credibly signal her strength: low-valuation seller types make revealing offers early in the game, while high-valuation seller types initially make nonserious offers until revealing later in the game. Cho (1990a) posited equilibria of finite-horizon seller-offer bargaining games with the properties that: (a) the seller's pricing rule is a separating strategy after every history; (b) equilibria satisfy a continuity property resembling trembling-hand perfection; (c) equilibria satisfy a monotonicity restriction on beliefs; and (d) equilibria are stationary.

However, both the Cramton (1984) and Cho (1990a) constructions ultimately exhibit an unfortunate property, when the seller and buyer distributions have the same supports and for short time intervals between offers. By the stationarity assumption, the lowest seller type is subject to the Coase Conjecture, earning a payoff arbitrarily close to zero. Meanwhile, higher seller types offer prices which always exceed their respective types. The lowest seller type thus faces a very strong incentive to mimic a higher seller type, breaking the equilibrium unless essentially all higher types encounter extremely long delays before trading. Thus, a No Trade Theorem holds: In the limit as the time interval between offers decreases toward zero, the ex ante expected probability of trade in these equilibria converges to zero (Ausubel and Deneckere, 1992b, Theorem 1).

Two other articles present plausible outcomes of dynamic bargaining games with two-sided incomplete information in which trade occurs to a substantial degree but which are inefficient compared to the constrained static optimum.[24] Cramton (1992) extends and analyzes the Admati-Perry (1987) extensive-form game to an environment with a continuum of types and two-sided incomplete information. The game begins with effectively a war of attrition between the seller and the buyer: there is a seller type $s(t)$ and a buyer type $b(t)$ who are each supposed to reveal themselves by making serious offers at time t. Thus, as the game unfolds without serious offers getting made, each party becomes more pessimistic about his counterpart's valuation. A serious offer—once made—fully reveals the offeror's type. The other player then either accepts the serious offer or further delays trade

[24] Perry (1986) analyzes an alternating-offer game with two-sided incomplete information about valuations, but where the cost of bargaining takes the form of a fixed cost per period rather than discounting. He establishes the existence of a unique sequential equilibrium when the players' fixed costs are unequal. When it is the turn of the player with the lower bargaining cost to make an offer, this player proposes essentially its monopoly price, which the other player accepts if it yields nonnegative utility. When it is the turn of the player with the higher bargaining cost to make an offer, this player leaves the game without making an offer. Thus, trade—if it occurs at all—occurs in the initial period. However, inefficiently little trade occurs compared to the constrained static optimum. Perry's game illustrates the principle that there is no possibility for signaling through delay when the incomplete information is about valuations but the bargaining cost is a fixed cost each period. Signaling requires the presence of an action which is relatively less costly for one type than another; in this game, the cost of delay is equal across all types.

so as to credibly convey his own type and, when trade occurs following both players' full revelation, it occurs at the complete-information price. Ausubel and Deneckere (1992b) consider the seller-offer bargaining game and construct a continuum of equilibria, all with the property that the seller's first serious offer reveals essentially all the information which she will ever reveal. One interesting equilibrium in this class is the "monopoly equilibrium": the seller fully reveals her type in the initial period by offering essentially the monopoly price relative to her valuation; and then follows a slowly-descending price path thereafter. This equilibrium is also ex ante efficient—provided that all of the weight is placed on the seller.

6 Empirical Evidence

Bargaining is pervasive in our economy. Thus, it is not surprising that there is a substantial empirical literature. However, only recently has this work sought to examine the data in light of strategic bargaining theories with private information.

Bargaining models with private information are especially well suited for empirical work, since a main feature of the data is the occurrence of costly disputes. These disputes arise naturally in models with incomplete information. However, private information models involve several challenges for empirical work. First, the models are often complex, making estimation difficult. Second, the results tend to be sensitive to the particular bargaining procedure, the source of private information, and the form of delay costs. In most empirical settings, the bargaining rules and the preferences of the parties cannot be fully identified. The researcher then may have too much freedom in selecting assumptions that "explain" particular facts. Finally, the theory predicts how ex post outcomes depend on realizations of private information, yet the researcher typically is unable to observe private information variables, even ex post.

We focus on one of the most prominent examples of bargaining—union contract negotiations—in understanding bargaining disputes.

Kennan and Wilson (1989) analyze attrition, screening, and signaling models, and contrast the theoretical predictions of these models with the main empirical features of strike data. They emphasize five empirical findings:

- Strikes are unusual, occurring in 10 to 20 percent of contract negotiations.
- The relationship between strike duration and wages is ambiguous. McConnell (1989) found that wages declined 3% per 100 days of strike in the U.S., but Card (1990) found no significant relationship between strike duration and wages.

- Strikes are more frequent in good times (Vroman 1989; Gunderson, Kervin, and Reid 1986), yet strike duration decreases in good times (Kennan 1985; Harrison and Stewart 1989).
- Strike activity varies across industries.
- Settlement rates tend to decline with strike duration (Kennan 1985; Harrison and Stewart 1989; Gunderson and Melino 1990).

In all of the models, strikes (or their absence) convey private information in a credible way. A key feature of attrition models is winner-take-all outcomes. In an attrition model, each side attempts to convince the other that it can last longer, so the other should concede the entire pie under negotiation. One side clearly wins at the expense of the other. In contrast, wage bargaining typically involves compromise. For this reason, we focus on screening and signaling models.

The standard setting assumes that the union is uncertain about the firm's willingness to pay. In this case, under either screening or signaling, the duration of the strike conveys information to the union about the firm's willingness to pay. A firm with a greater willingness to pay settles early at a high wage; whereas, a firm with a low willingness to pay endures a strike in order to convince the union to accept a low wage. A documentary film, Final Offer, of the 1984 negotiations between GM Canada and the UAW provides anecdotal evidence for this explanation for strikes. Early in the strike the union leaders are discussing whether they should accept GM's last offer. One says, "You might convince me that that's all there is after a month, but not after five days." Another says, "If they think it will take a short strike to convince workers to accept, they're wrong."

Screening and signaling models share several features: (1) strike incidence and strike duration increase with uncertainty over private information variables, and (2) wages fall with strike duration. However, there are important differences in wages and strike activity.

The standard screening model assumes that the union makes a sequence of declining wage demands, with each demand chosen optimally given beliefs about the firm's willingness to pay and the firm's acceptance and offer strategy. A critical assumption is that the firm employs a stationary acceptance strategy. At every point in the negotiation, a firm with value v accepts any wage demand below $w(v)$. Most importantly, this assumption means that the firm's acceptance rule cannot depend on the rate of concession by the union. This greatly limits the equilibrium set, assuring that all equilibria satisfy the Coase (1972) conjecture. As the time between offers shrinks, the union loses its bargaining power and makes offers that are close to the Rubinstein wage between the union and the lowest-value firm. Strike duration falls to zero and strike incidence increases to one, but the convergence is slow. Screening then has the property that wages, strike incidence, and strike duration all depend critically on the period over which the union can commit to a wage demand. Kennan and Wilson (1989) argue that the Coase conjecture may explain why in boom times strikes are more frequent but shorter. This would follow if the union has a shorter commitment period in boom times; however, it is not clear why the time between offers would vary with the business cycle.

One potential difficulty with the screening model is that, because of the Coase property, strike durations must be short when the commitment period is short. In the U.S., mean strike durations are about 40 days. If offers can be made every day, then the standard screening model may predict strikes that are too short given plausible interest rates and levels of uncertainty. Hart (1989) provides an explanation. If bargaining costs are low initially, but then increase at some point during the strike, say when inventories run out, then strikes can be much longer. Another explanation is given by Vincent (1989). If the parties' valuations are interdependent, then strikes of significant duration can occur even as the time between offers goes to zero.

The signaling model arises when the time between offers is endogenous (Admati and Perry, 1987). Then the informed party (the firm) has an incentive to delay making an offer until after a sufficient time has passed to credibly reveal its private information. The critical assumption here is that the uninformed party (the union) is unable to make a counteroffer while it is waiting for the firm to make an offer. Aside from the union's initial demand, all settlements are ex post fair, in that the wage is the full-information Rubinstein (1982) wage. The union's initial demand is chosen to balance the cost of delay and the terms of settlement. This initial demand is accepted by the firm if its willingness to pay is sufficiently high. Otherwise the firm makes a counteroffer after waiting long enough to make the Rubinstein wage credible. Signaling and screening can be compared along a number of dimensions:

- Screening outcomes depend critically on the minimum time between offers; signaling outcomes are insensitive to the minimum time between offers.
- Screening outcomes strongly favor the informed party (the firm); signaling outcomes are roughly ex post fair. Hence, wages are higher under signaling and are more sensitive to the firm's private information.
- Dispute incidence and dispute durations are higher under signaling. Indeed, dispute incidence is always greater than 50% in the standard signaling model. However, introducing a fixed cost of initiating a strike can lead to any level of strike incidence.

Cramton and Tracy (1992) emphasize that the union has multiple threats. The union can strike or the union can holdout, putting pressure on the firm while continuing to work. Holdouts take the form of a slowdown, work-to-rule, sick-out, or other in-plant action. From the union's point of view, holdouts have two advantages: (1) workers are paid according to the expired contract, and (2) workers cannot be replaced. The union selects the threat, strike or holdout, that gives it the highest payoff. Since the desirability of each threat depends on observable factors, modeling this threat choice is important to understanding key features of the data. When striking is the only threat, then strike incidence depends essentially on the degree of uncertainty; whereas, with multiple threats strike incidence can vary as the composition of disputes changes with the attractiveness of each threat. For example, holdouts are more desirable when the current wage is high, and strikes are more desirable when unemployment is low and the workers have better outside options.

In Cramton and Tracy (1992), a union and a firm are bargaining over the wage to be paid over the next contract period. The union's reservation wage is common knowledge. The firm's value of the labor force is private information.

Bargaining begins with the union selecting a threat, either holdout or strike, which applies until a settlement is reached. In the holdout threat, the union is paid the current wage under the expired contract. There is some inefficiency associated with holdout. An outcome of the bargaining specifies the time of agreement, the contract wage at the time of agreement, and the threat before agreement. Following the union's threat choice, the union and firm alternate wage offers, with the union making the initial offer. The time between offers is endogenous.

The equilibrium takes a simple form. If the current wage is sufficiently low, the union decides to strike; otherwise, the union holds out. A second indifference level is determined by the union's initial offer. The firm accepts the union's initial offer if its valuation is above the indifference level, and otherwise rejects the offer and makes a counteroffer after sufficient time has past to credibly signal the firm's value.

A primary result is that dispute activity increases with uncertainty about private information. Tracy (1986, 1987) tests this basic result by using stock price volatility as a proxy for the amount of uncertainty in contract negotiations. With U.S. data, he finds that strike incidence and strike duration increase with greater relative volatility.

Cramton and Tracy (1994a) fit the parameters of the model to match the main features of the U.S. data from 1970 to 1989. They also estimate dispute incidence and dispute composition. Consistent with the theory, strike incidence increases as the strike threat becomes more attractive, because of low unemployment or a real wage drop over the previous contract. However, the model performs less well in the 1980s than in the 1970s, suggesting a structural change in the post-1981 period. One explanation for a shift is an increase in the use of replacement workers following President Reagan's firing of striking air traffic controllers. Indeed, there was a shift away from strikes and towards holdouts in the 1980s.

Cramton and Tracy (1998) investigate the extent to which the hiring of replacement workers can account for these changes. They build a model in which a firm considers the replacement option because it improves the firm's strike payoff relative to the union's, resulting in a lower wage. However, a firm must balance this improvement in the terms of trade with the cost of replacement. A firm only uses replacements if its cost of replacement is sufficiently low. The union, anticipating the possibility of replacement, lowers its wage demand in the strike threat in order to reduce the probability of replacement. This risk of replacement, then, reduces the attractiveness of the strike threat, making it more likely that the union adopts the holdout threat at the outset of negotiations. For all large U.S. strikes in the 1980s, the likelihood of replacement is estimated. Consistent with the model, the composition of disputes shifts away from strikes as the predicted risk of replacement increases. Hence, a ban on the use of replacement workers should increase strike activity. Moreover, a ban on replacement increases uncertainty, since replacement effectively truncates the firm's distribution of willingness to pay (Kennan and Wilson 1989).

The Canadian data provide an opportunity to test this theory. Quebec instituted a ban on replacements in 1977, and British Columbia and Ontario introduced a similar ban in 1993. Gunderson, Kervin, and Reid (1989) find that strike incidence does increase with a ban on replacements, and Gunderson and Melino (1990) find strikes are longer after a ban. Budd (1996) and Cramton, Gunderson, and Tracy (1999) examine the effect of a ban on replacement workers on wages and strike activity. Budd does not find significant effects from the ban using a sample of single province contracts in manufacturing from 1965-1985. In contrast, with a larger sample of contract negotiations from 1967-1993, Cramton, Gunderson, and Tracy find that prohibiting the use of replacement workers during strikes is associated with significantly higher wages, and more frequent and longer strikes.

Predictions of the bargaining models are sensitive to how threat payoffs change over time. Hart (1989) shows that strike durations are much longer in a screening model when strike costs increase sharply when a crunch point is reached (say inventories run out). Cramton and Tracy (1994b) consider time-varying threats within a signaling model. Strike payoffs change as replacement workers are hired, as strikers find temporary jobs, and as inventories or strike funds run out. The settlement wage is largely determined from the long-run threat, rather than the short-run threat. As a result, if dispute costs increase in the long run, then dispute durations are longer and wages decline more slowly during the short run. Allowing time-varying threats helps explain empirical results. Settlement rates are lower during periods of eligibility for unemployment insurance (Kennan 1980). Strike durations are longer during business downturns (Kennan 1985; Harrison and Stewart 1989). Wages might not decrease with strike durations (Card 1990). Moreover, the theory can help explain the costly actions firms and unions take to influence threat payoffs.

An important feature of union contract negotiations is that they do not occur in isolation. Information from one contract negotiation may be linked with other contract negotiations within the same industry. Kuhn and Gu (1996) interpret holdouts in this way. In their theory, holdouts are used as a delaying tactic to get information about other bargaining outcomes in the same industry. When private information is correlated among bargaining pairs, there is an incentive to holdout, since one bargaining pair benefits from information revealed in the negotiation of another pair. Three predictions stem from this theory: (1) holdouts should increase when more bargaining pairs negotiate concurrently, (2) there should be a clustering of holdout durations within an industry, and (3) holdouts ending later are less apt to end in strikes. A panel of Canadian manufacturing contract negotiations from 1965 to 1988 support these predictions. A further implication of the linked information is that strike incidence can be reduced to the extent that private information is revealed in related contract negotiations. Kuhn and Gu (1995) find support for this hypothesis.

In addition to within-industry links, contracts are linked over time. Today's negotiation is just one in a sequence of negotiations between the union and the firm. The current negotiation affects the next negotiation in two ways: a wage linkage and an information linkage. The wage linkage is as in Cramton and Tracy (1992). The current wage is the starting point for negotiations and determines the attractiveness of striking versus holding out. An information linkage arises when the private information between contracts is correlated. Kennan (1995) studies a screening model of repeated negotiations where the firm's willingness to pay follows a Markov process. One implication of this model is a rachet effect. A firm is more hesitant to give in today, knowing that doing so will worsen its position in the next negotiation. More importantly, Kennan's model of repeated negotiation can explain some of the observed links between prior and current contract negotiations. For example, Card (1988, 1990) finds that strike incidence is higher after a short strike in the prior negotiation, and lower after either no strike or a long strike in the prior negotiation.

7 Experimental Evidence

Strategic theories of bargaining with private information only recently have been evaluated in the experimental laboratory. The advantage of an experimental test of the theory, compared with an empirical test, is that the experimenter is able to observe the distribution and realizations of private information. The power of empirical tests is limited because the parties' degree of uncertainty must be estimated indirectly from the data, under the assumption that the theory is true. This has led most researchers to test other empirical implications of the model, such as the slope of the concession function. The experimenter, on the other hand, can construct an environment that conforms much more closely to the theoretical setting. In this way, less ambiguous tests of the theory can be performed. Unfortunately, even in tightly controlled experiments, some ambiguity will remain, since the subjects may have relevant private information about their preferences that the experimenter is not privy to.[25]

Most of the experimental work on strategic bargaining has focused on testing dynamic models with full information[26] or static models with private information.[27] Much could be learned by considering dynamic bargaining with private information. By introducing private information into a dynamic bargaining environment, we are able to observe how uncertainty influences the incidence and duration of disputes. This has been the focus of much of the theoretical and empirical work, and yet few experimental tests have been done.

Bibliography

Admati, Anat and Motty Perry (1987), "Strategic Delay in Bargaining," Review of Economic Studies, 54, 345-364.

Akerlof, George A. (1970), "The Market for 'Lemons:' Quality Uncertainty and the Market Mechanism," Quarterly Journal of Economics, 84, 488-500.

Ausubel, Lawrence M. (2002), "A Mechanism Generalizing the Vickrey Auction," Econometrica, forthcoming.

Ausubel, Lawrence M. and Raymond J. Deneckere (1989a), "Reputation in Bargaining and Durable Goods Monopoly," Econometrica, 57, 511-532.

Ausubel, Lawrence M. and Raymond J. Deneckere (1989b), "A Direct Mechanism Characterization of Sequential Bargaining with One-Sided Incomplete Information," Journal of Economic Theory, 48, 18-46.

[25] This point is emphasized by Forsythe, Kennan and Sopher (1991) and Ochs and Roth (1989).

[26] Binmore, Shaked and Sutton (1985, 1988, 1989), Neelin, Sonnenschein and Spiegel (1988), and Ochs and Roth (1989).

[27] Forsythe, Kennan and Sopher (1991) and Radner and Schotter (1989).

Ausubel, Lawrence M. and Raymond J. Deneckere (1992a), "Bargaining and the Right to Remain Silent," Econometrica, 60, 597-626.

Ausubel, Lawrence M. and Raymond J. Deneckere (1992b), "Durable Goods Monopoly with Incomplete Information," Review of Economic Studies, 59, 795-812.

Ausubel, Lawrence M. and Raymond J. Deneckere (1993a), "Efficient Sequential Bargaining," Review of Economic Studies, 60, 435-462.

Ausubel, Lawrence M. and Raymond J. Deneckere (1993b), "A Generalized Theorem of the Maximum," Economic Theory, 3, 99-107.

Ausubel, Lawrence M. and Raymond J. Deneckere (1994), "Separation and Delay in Bargaining," mimeo, University of Maryland and University of Wisconsin-Madison.

Ausubel, Lawrence M. and Raymond J. Deneckere (1998), "Bargaining and Forward Induction," mimeo, University of Maryland and University of Wisconsin-Madison.

Ayres, Ian and Eric Talley (1995), "Solomonic Bargaining: Dividing a Legal Entitlement to Facilitate Coasean Trade," Yale Law Journal, 104, 1027-1117.

Bikhchandani, Sushil (1992), "A Bargaining Model with Incomplete Information," Review of Economic Studies, 59, 187-204.

Binmore, Ken, Martin J. Osborne, and Ariel Rubinstein (1992), "Noncooperative Models of Bargaining," in R.J. Aumann and S. Hart (eds) Handbook of Game Theory, Vol. 1, 179-225, North Holland: Amsterdam.

Binmore, Ken, Avner Shaked and John Sutton (1985), "Testing Noncooperative Bargaining Theory: A Preliminary Study," American Economic Review, 75, 1178-1180.

Binmore, Ken, Avner Shaked, and John Sutton (1988), "A Further Test of Noncooperative Bargaining Theory: Reply," American Economic Review, 78, 837-840.

Binmore, Ken, Avner Shaked, and John Sutton (1989), "An Outside Option Experiment," Quarterly Journal of Economics, 104, 753-770.

Budd, John W. (1996), "Canadian Strike Replacement Legislation and Collective Bargaining: Lessons for the United States," Industrial Relations, 35, 245-60.

Card, David (1988), "Longitudinal Analysis of Strike Activity," Journal of Labor Economics, 6, 147-176.

Card, David (1990), "Strikes and Wages: A Test of a Signalling Model," Quarterly Journal of Economics, 105, 625-660.

Chatterjee, Kalyan and William Samuelson (1983), "Bargaining Under Incomplete Information," Operations Research, 31, 835-851.

Cho, In-Koo (1990a), "Uncertainty and Delay in Bargaining," Review of Economic Studies, 57, 575-596.

Cho, In-Koo (1990b), "Characterization of Equilibria in Bargaining Models with Incomplete Information," mimeo, University of Chicago.

Cho, In-Koo and David M. Kreps (1987), "Signalling Games and Stable Equilibria," Quarterly Journal of Economics, 102(2), 179−222.

Cho, In-Koo and Joel Sobel (1990), "Strategic Stability and Uniqueness in Signaling Games," Journal of Economic Theory, 50(2), 381−413.

Coase, Ronald H. (1960), "The Problem of Social Cost," Journal of Law and Economics, 3, 1-44.

Coase, Ronald H. (1972), "Durability and Monopoly," Journal of Law and Economics, 15, 143-149.

Cramton, Peter (1984), "Bargaining with Incomplete Information: An Infinite-Horizon Model with Continuous Uncertainty," Review of Economic Studies, 51, 579-593.

Cramton, Peter (1985), “Sequential Bargaining Mechanisms,” in Alvin Roth (ed), Game Theoretic Models of Bargaining, Cambridge, England: Cambridge University Press.

Cramton, Peter (1992), “Strategic Delay in Bargaining with Two-Sided Uncertainty,” Review of Economic Studies, 59, 205-225.

Cramton, Peter, Robert Gibbons, and Paul Klemperer (1987), “Dissolving a Partnership Efficiently,” Econometrica, 55, 615-632.

Cramton, Peter, Morley Gunderson, and Joseph Tracy (1999), “The Effect of Collective Bargaining Legislation on Strikes and Wages,” Review of Economics and Statistics, forthcoming.

Cramton, Peter and Joseph Tracy (1992), “Strikes and Holdouts in Wage Bargaining: Theory and Data,” American Economic Review, 82, 100-121.

Cramton, Peter and Joseph Tracy (1994), “Wage Bargaining with Time-Varying Threats,” Journal of Labor Economics, 12, 594-617.

Cramton, Peter and Joseph Tracy (1994), “The Determinants of U.S. Labor Disputes,” Journal of Labor Economics, 12, 180-209.

Cramton, Peter and Joseph Tracy (1998), “The Use of Strike Replacements in Union Contract Negotiations: the U.S. Experience, 1980-1989,” Journal of Labor Economics, 16, 667-701.

Crawford, Vincent P. (1982), “A Theory of Disagreement in Bargaining,” Econometrica, 50, 607-637.

Cremer, Jacques and Richard McLean (1988), “Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions,” Econometrica, 56, 1247-1258.

Dasgupta, Partha and Eric Maskin (2000), “Efficient Auctions,” Quarterly Journal of Economics, 115, 341-388.

Deneckere, Raymond J. (1992), “A Simple Proof of the Coase Conjecture,” mimeo, Northwestern University.

Deneckere, Raymond J. and Meng-Yu Liang (1999), “Bargaining with Interdependent Values,” mimeo, University of Wisconsin-Madison.

Evans, Robert (1989), “Sequential Bargaining with Correlated Values,” Review of Economic Studies, 56(4), 499-510.

Farrell, Joseph and Robert Gibbons (1989), “Cheap Talk Can Matter in Bargaining,” Journal of Economic Theory, 48, 221-237.

Fernandez, Raquel and Jacob Glazer (1991), “Striking for a Bargain Between Two Completely Informed Agents,” American Economic Review, 81, 240-252.

Forsythe, Robert, John Kennan, and Barry Sopher (1991), “An Experimental Analysis of Strikes in Bargaining Games with One-Sided Private Information,” American Economic Review, 81, 253-278.

Fudenberg, Drew, David K. Levine and Jean Tirole (1985), “Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information,” in Alvin Roth (ed), Game Theoretic Models of Bargaining, Cambridge, England: Cambridge University Press.

Gresik, Thomas A. (1991a), “Efficient Bilateral Trade with Statistically Dependent Beliefs,” Journal of Economic Theory, 53, 199-205.

Gresik, Thomas A. (1991b), “Ex Ante Efficient, Ex Post Individually Rational Trade,” Journal of Economic Theory, 53, 131-145.

Gresik, Thomas A. (1991c), “Ex Ante Incentive Efficient Trading Mechanisms without the Private Valuation Restriction,” Journal of Economic Theory, 55, 41-63.

Gresik, Thomas A. and Mark A. Satterthwaite (1989), “The Rate at Which a Simple Market Converges to Efficiency as the Number of Traders Increases: An Asymptotic Result for Optimal Trading Mechanisms,” Journal of Economic Theory, 48, 304-332.

Grossman, Sanford J. and Motty Perry (1986a), “Perfect Sequential Equilibrium,” Journal of Economic Theory, 39(1), 97-119.

Grossman, Sanford J. and Motty Perry (1986b), “Sequential Bargaining Under Asymmetric Information,” Journal of Economic Theory, 39(1), 120-154.

Gul, Faruk and Hugo Sonnenschein (1988), “On Delay in Bargaining with One-Sided Uncertainty,” Econometrica, 56, 601-612.

Gul, Faruk, Hugo Sonnenschein and Robert Wilson (1986), “Foundations of Dynamic Monopoly and the Coase Conjecture,” Journal of Economic Theory, 39, 155−190.

Gunderson, Morley, John Kervin, and Frank Reid (1986), “Logit Estimates of Strike Incidence from Canadian Contract Data,” Journal of Labor Economics, 4, 257-276.

Gunderson, Morley, John Kervin, and Frank Reid (1989), “The Effect of Labour Relations Legislation on Strike Incidence,” Canadian Journal of Economics, 22, 779-794.

Gunderson, Morley and Angelo Melino (1990), “Estimating Strike Effects in a General Model of Prices and Quantities,” Journal of Labor Economics, 5, 1-19.

Gunderson, Morley and Angelo Melino (1990), “The Effects of Public Policy on Strike Duration,” Journal of Labor Economics, 8, 295-316.

Haller, Hans and Steinar Holden (1990), “A Letter to the Editor on Wage Bargaining,” Journal of Economic Theory, 52, 232-236.

Harrison, Alan and Mark Stewart (1989), “Cyclical Fluctuations in Strike Durations,” American Economic Review, 79, 827-841.

Hart, Oliver D. (1989), “Bargaining and Strikes,” Quarterly Journal of Economics, 104, 25-44.

Hart, Oliver D. and Jean Tirole (1988), “Contract Renegotiation and Coasian Dynamics,” Review of Economic Studies, 55, 509-540.

Jehiel, Philippe and Benny Moldovanu (2001), “Efficient Design with Interdependent Valuations,” Econometrica, forthcoming.

Kennan, John (1980), “The Effect of Unemployment Insurance Payments on Strike Duration,” Unemployment Compensation: Studies and Research, 2, 467-483.

Kennan, John (1985), “The Duration of Contract Strikes in US Manufacturing,” Journal of Econometrics, 28, 5-28.

Kennan, John (1995), “Repeated Contract Negotiations with Private Information,” Japan and the World Economy, 7, 447-472.

Kennan, John (1997), “Repeated Bargaining with Persistent Private Information,” mimeo, Department of Economics, University of Wisconsin-Madison.

Kennan, John and Robert Wilson (1989), “Strategic Bargaining Models and Interpretation of Strike Data,” Journal of Applied Econometrics, 4, S87-S130.

Kennan, John and Robert Wilson (1993), “Bargaining with Private Information,” Journal of Economic Literature, 31, 45-104.

Kohlberg, Elon and Jean-Francois Mertens (1986), “On the Strategic Stability of Equilibria,” Econometrica, 54, 1003-1037.

Krishna, Vijay and Motty Perry (1997), “Efficient Mechanism Design,” Working Paper, Penn State University.

Kuhn, Peter and Wulong Gu (1995), “The Economics of Relative Rewards: Sequential Wage Bargaining,” Working Paper, McMaster University.

Kuhn, Peter and Wulong Gu (1996), “A Theory of Holdouts in Wage Bargaining,” Working Paper, McMaster University.

Makowski, Louis and Claudio Mezzetti (1993), “The Possibility of Efficient Mechanisms for Trading an Indivisible Object,” Journal of Economic Theory, 59, 451-465.

Maskin, Eric and Jean Tirole (1994), “Markov Perfect Equilibrium,” mimeo, Harvard University.

Matsuo, Toshihide (1989), “On Incentive Compatible, Individually Rational, and Ex-Post Efficient Mechanisms for Bilateral Trading,” Journal of Economic Theory, 49(1), 189-194.

McAfee, R. Preston (1992), “Amicable Divorce: Dissolving a Partnership with Simple Mechanisms,” Journal of Economic Theory, 56, 266-293.

McAfee, Preston R. and Philip Reny (1992), “Correlated Information and Mechanism Design,” Econometrica, 60, 395-422.

McConnell, Sheena (1989), “Strikes, Wages, and Private Information,” American Economic Review, 79, 801-815.

Mirrlees, James A. (1971), “An Exploration in the Theory of Optimal Taxation,” Review of Economic Studies, 38, 175-208.

Mookherjee, Dilip, and Stefan Reichelstein (1992), “Dominant Strategy Implementation of Bayesian Incentive Compatible Allocation Rules,” Journal of Economic Theory, 56, 378-399.

Myerson, Roger B. (1981), “Optimal Auction Design,” Mathematics of Operations Research, 6, 58-73.

Myerson, Roger B. (1985), “Analysis of Two Bargaining Problems with Incomplete Information,” in Game Theoretic Models of Bargaining, A. Roth (ed.), Cambridge University Press, London, 115−147.

Myerson, Roger B. and Mark A. Satterthwaite (1983), “Efficient Mechanisms for Bilateral Trading,” Journal of Economic Theory, 28, 265-281.

Neelin, Janet, Hugo Sonnenschein, and Matthew Spiegel (1988), “A Further Test of Nonccoperative Bargaining Theory,” American Economic Review, 78, 824-836.

Ochs, Jack and Alvin E. Roth (1989), “An Experimental Study of Sequential Bargaining,” American Economic Review, 89, 355-384.

Osborne, Martin J. and Ariel Rubinstein (1990), Bargaining and Markets, Boston: Academic Press.

Perry, Motty (1986), “An Example of Price Formation in Bilateral Situations: A Bargaining Model with Incomplete Information,” Econometrica, 54(2), 313-21.

Perry, Motty and Philip J. Reny (1993), “A Non-Cooperative Bargaining Model with Strategically Timed Offers,” Journal of Economic Theory, 59(1), 50-77.

Perry, Motty and Philip J. Reny (1998), “An Ex-Post Efficient Auction,” mimeo, University of Pittsburgh.

Radner, Roy and Andrew Schotter (1989), “The Sealed-Bid Mechanism: An Experimental Study,” Journal of Economic Theory, 48, 179-220.

Riley, John and Richard Zeckhauser (1983), “Optimal Selling Strategies: When to Haggle, When to Hold Firm,” Quarterly Journal of Economics, 98(2), 267-289.

Rubinstein, Ariel (1982), “Perfect Equilibrium in a Bargaining Model,” Econometrica, 50, 97-109.

Rubinstein, Ariel (1985a), “A Bargaining Model with Incomplete Information about Time Preferences,” Econometrica, 53, 1151-1172.

Rubinstein, Ariel (1985b), “Choice of Conjectures in a Bargaining Game with Incomplete Information,” in Game Theoretic Models of Bargaining, op. cit., 99−114.

Rubinstein, Ariel (1987), “A Sequential Strategic Theory of Bargaining,” in Advances in Economic Theory, T.

Bewley (ed.), Cambridge University Press, London, 197−224.

Rustichini, Aldo, Mark A. Satterthwaite, and Steven R. Williams (1994), “Convergence to Efficiency in a Simple Market with Incomplete Information,” Econometrica, 62, 1041-1064.

Sákovics, Jozsef (1993), “Delay in Bargaining Games with Complete Information,” Journal of Economic Theory, 59(1), 78-95.

Samuelson, William (1984), “Bargaining Under Asymmetric Information,” Econometrica, 52, 995-1005.

Samuelson, William (1985), “A Comment on the Coase Theorem,” in Alvin Roth (ed), Game Theoretic Models of Bargaining, Cambridge, England: Cambridge University Press.

Satterthwaite, Mark, A and Steven R. Williams (1989), “The Rate of Covergence to Efficiency in the Buyer’s Bid Double Auction as the Market Becomes Large,” Review of Economic Studies, 56, 477-498.

Sobel, Joel and Ichiro Takahashi (1983), “A Multi-Stage Model of Bargaining,” Review of Economic Studies, 50, 411−426.

Stokey, Nancy L. (1979), “Intertemporal Price Discrimination,” Quarterly Journal of Economics, 93, 355-371.

Stokey, Nancy L. (1981), “Rational Expectations and Durable Goods Pricing,” Bell Journal of Economics, 12, 112-128.

Tracy, Joseph S. (1986), “An Investigation into the Determinants of U. S. Strike Activity,” American Economic Review, 76, 423-436.

Tracy, Joseph S. (1987), “An Empirical Test of an Asymmetric Information Model of Strikes,” Journal of Labor Economics, 5, 149-173.

Vincent, Daniel R. (1989), “Bargaining with Common Values,” Journal of Economic Theory, 48, 47-62.

Vincent, Daniel R. (1998), “Repeated Signalling Games and Dynamic Trading Relationships,” International Economic Review, 39(2), 275-93.

Vroman, Susan B. (1989), “A Longitudinal Analysis of Strike Activity in U.S. Manufacturing: 1957-1984,” American Economic Review, 79, 816-826.

Williams, Steven R. (1990), “The Transition from Bargaining to a Competitive Market,” American Economic Review, 80, 227-231.

Williams, Steven R. (1991), “Existence and Convergence of Equilibria in the Buyer’s Bid Double Auction,” Review of Economic Studies, 58, 351-374.

Williams, Steven R. (1999), “A Characterization of Efficient, Bayesian Incentive Compatible Mechanisms,” Economic Theory, 14(1), 155-180.

Wilson, Robert (1985), “Incentive Efficiency of Double Auctions,” Econometrica, 53, 1101-1116.