Health, Investment in Health, and Occupational Choice

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This paper presents two models of investment in health which explicitly recognize the random nature of illness and death. The first model examines life-cycle behavior of investment and health capital when the motive for investing in health is to decrease the probability of illness. In the second model the individual invests in health through his choice of occupation. This determines the extent of his exposure to a pollutant, such as asbestos, which increases the probability of death. The model examines how exposure to pollution should vary with age and predicts how workers should respond to information about occupational dangers.

I. Introduction

In recent years it has been recognized that expenditures on medical services, nutrition, and exercise can be viewed as investments in health capital and analyzed using the tools of capital theory. This approach has enabled economists to derive propositions about the pattern of medical expenditures over an individual's lifetime and to describe the behavior of health capital over the life cycle. In addition, it has provided a means of explaining observed cross-sectional differences in medical expenditures. Grossman (1972), for example, has developed a model in which health is viewed as a capital stock which yields an output of "healthy days." Individuals may invest in health by combining time (e.g., for doctor's visits) with purchased inputs (e.g., medical services). The incentive for investing in health is that by increasing the health stock the individual increases the amount of time available for earning income or for producing consumption goods.

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In the present paper, investment in health is approached from a slightly different viewpoint. Specifically, I assume that individuals invest in health not to receive a monetary return but to avoid the disutility associated with being ill.[1] Since individuals are usually unable to enjoy life fully when they are ill, illness will be treated as a temporary interruption in the individual's utility stream. Death, which is also affected by the stock of health capital, can be regarded as a permanent interruption of the utility stream.

A second point of departure is that the relation between health capital and illness is treated here as random rather than as deterministic. Whether one is well or ill depends, after all, on random events—changes in climate, exposure to viruses and germs—as well as upon the size of one's health stock. Formally, I shall assume that illness occurs whenever the health stock, H, falls below some critical level $\bar{H}$, where $\bar{H}$ is a random variable. The individual thus increases the probability of being well by maintaining a high stock of health capital but cannot guarantee that illness will not occur. An equivalent way of expressing this is to say that at each instant of the individual's life one of two states—"ill" or "not ill"—must occur. The probability of either state obtaining depends on the stock of health capital and can be determined indirectly by investing in health.

Since individuals are not fully able to enjoy life when they are ill, illness is assumed temporarily to interrupt the individual's utility stream; that is, if the state "ill" occurs the individual receives the same utility he would receive if he consumed nothing. Thus psychically, if not literally, illness is equivalent to zero consumption.

Aside from possible theoretical appeal, this treatment of illness enables one to distinguish between two very different types of medical expenditure: preventative medical expenditure and expenditure in the event of illness. This distinction is an important one since the two types of investment may behave quite differently over the life cycle. The present model focuses on preventative expenditure on health, which is nonstochastic, and shows that this form of investment may decrease as the individual ages and the period over which returns are received decreases. Expenditure in the event of illness, on the other hand, is a random variable which is positively related to the occurrence of illness. Although the model does not deal explicitly with this type of investment, it is shown that the fraction of time spent ill increases as the individual ages, hence so should expenditure necessitated by illness.

A second advantage of our approach to investment in health is that it provides a model which, with suitable modification, can be used to analyze a more subtle form of investment in health capital. Casual

[1] Grossman emphasizes that good health should be regarded as a consumption as well as an investment good; however, in the principal case which he analyzes, health services do not enter the utility function directly.

observation suggests that there is often a trade-off between high wages and job safety, especially in unskilled jobs. Hence, in order to earn a higher income an individual must often work in occupations which are damaging to his health and may even result in his death. This implies that the choice of an occupation is also a form of investment in health.

In recent years attention has been focused on a particular class of jobs which are injurious to the health, namely, jobs involving industrial pollutants.[2] The more well-publicized of these pollutants include asbestos, kepone, and nuclear radiation; however, the problem is considerably larger, for the U.S. Food and Drug Administration lists some 25,000 substances as potentially hazardous to the health. Like asbestos, most industrial pollutants are absorbed gradually, in proportion to the number of hours worked, and accumulate inside the worker's body. The interesting feature of the process is that the effects of these pollutants may not be felt for many years; however, once enough of the pollutant has accumulated, death will result. Unfortunately, what constitutes a lethal dose of many pollutants must be considered uncertain. The problem facing the worker is therefore one of trading higher wages for an increase in the probability of death.

The existence of jobs involving industrial pollutants raises several interesting questions. First, if an individual is faced with a choice between a job such as coal mining which is known to be dangerous and a safer but lower-paying occupation, how should he optimally behave? That is, by how much should he increase the probability of death in order to earn a higher income, and how should employment in the risky occupation vary over the life cycle? Second, if an individual has been working for years in an occupation which is suddenly discovered to be dangerous to the health, should he continue working, given that he has already accumulated a large stock of the pollutant, or should he change his occupation and allow the pollutant to decay?

The answers to these questions, which are developed in Section III, are somewhat surprising. It might seem, for example, that if an individual chooses to work at all in a risky occupation, he should do so when he is young, switching to a healthier occupation as he grows older so that his pollution stock may decrease. Under plausible conditions, however, it may be shown that the optimal pollution stock never decreases and that some exposure to the pollutant is always called for. Similarly, when an individual who has inadvertently accumulated a large stock of pollutant suddenly discovers that the substance is dangerous to his health, optimal

[2] A second type of job-related health risk is the risk of industrial accidents. Unlike pollutants, whose effects persist long after employment has ended, these risks are of a temporary nature only and may be thought of as increasing the probability of death while the individual is at work. Since the problem of industrial accidents has been dealt with by Thaler and Rosen (1975), I shall not deal with that problem here.

behavior may consist of further increasing the stock of the pollutant rather than switching to a less-risky job.

These conclusions are based on a model, set forth in Section III, which is formally analogous to the model of conventional investment in health presented in Section II. Results are summarized in Section IV.

II. A Model of Investment in Health Capital

A. The Model

Let us assume that at each instant an individual receives an income of $\bar{Y}$ which he divides between expenditure on a consumption good, C, and investment in health capital. This assumption ignores the possibility that the individual invests in human or nonhuman capital; however, it permits us to focus on investment in health and reduces the model to a single state variable. If investment in health has a constant marginal cost of c and if the consumption good has a price of v per unit, then the instantaneous budget constraint may be written

$$\bar{Y} = cI + vC, \tag{1}$$

or equivalently,

$$C = Y - pI, \tag{2}$$

where $Y \equiv \bar{Y}/v$ and $p \equiv c/v$.[3] Expenditure on consumption increases the individual's utility, provided that he is well, while investment in health increases the stock of health capital according to the relation

$$\dot{H} = I - \delta H. \tag{3}$$

Equation (3) reflects the fact that the health stock decays over time as the individual ages. Initially it is assumed that the rate of decay is constant throughout the individual's lifetime; however, the possibility that δ increases over time is considered below.

The size of the health stock is important since it determines at any instant whether the individual is well or ill. Specifically, the event "illness" occurs whenever the health stock H_t falls below a critical sickness level $\bar{H}_t$, which is assumed to be random. If $\bar{H}_t$ is viewed as the level of germs and viruses to which the individual is exposed, and if H_t is interpreted as resistance to disease, then this assumption characterizes fairly accurately

[3] Another way of expressing this is to say that at each instant the individual has a fixed amount of labor which he can devote to earning income, producing consumption goods, or producing investment in health. Consumption and investment in health are produced, as in Grossman (1972), by combining labor with purchased inputs. As long as the production functions for C and I are homogeneous of degree one, this production-oriented framework is equivalent to the assumptions made above.

minor illnesses such as colds, viruses, and influenza. In keeping with this interpretation of illness it is assumed that sickness lasts only momentarily and does not reduce the stock of health capital.[4]

The random nature of $\bar{H}_t$ is captured in the assumption that there exists a density function $f(\bar{H})$ over the interval $[0, \infty)$ from which a new $\bar{H}$ is drawn at each t. Successive drawings are assumed independent so that $\bar{H}_t$ is independently and identically distributed for all t. In view of these assumptions, the probability of the state "not ill" may be written

$$F(H_t) = \int_0^{H_t} f(\bar{H})\, d\bar{H},$$

while the probability of being ill is given by $1 - F(H_t)$.

An important feature of the model is the way in which illness affects the individual's utility. If at any t the state "well" occurs, then the individual receives utility from consumption according to the function $U(C)$,

$$U'(C) > 0,\ U''(C) < 0,\ U'(0) = \infty,\ U(0) = 0. \tag{4}$$

If the state "illness" occurs, utility is given by the function $V(C) \equiv 0$, all C. Although the individual may literally consume some of the good C when he is ill, it is assumed that the disutility of illness renders this psychically equivalent to zero consumption.[5]

Since illness is a random event, instantaneous expected utility can be written, by substitution from (2), as

$$EU_t = F(H)U(Y - pI). \tag{5}$$

Given an initial stock of health H_0, the problem facing the individual is to choose a path I_t and a value H_T to maximize

$$\int_0^T e^{-\rho t} F(H)U(Y - pI)\, dt \tag{6}$$

[4] Since the model applies only to minor illnesses which do not affect the stock of health capital, it is reasonable that the individual plans preventative health expenditure at $t = 0$ for the remainder of his life. If one wanted to model major illness, this could be accomplished by having illness (a random event) reduce the size of the health stock. In this case H_t would be a random variable, and the individual would determine his investment in health at each t, based on the most recently observed value of H_t.

[5] The assumption that $V(C) \equiv 0$ in the event of illness does not imply that the individual will plan to consume nothing when he is ill. For further discussion about this point the reader is referred to the beginning of Section IIB. One might criticize my treatment of illness on the grounds that it does not distinguish among illnesses according to their severity. This can be corrected, however, by assuming that the individual receives utility from services ϕH_t which are proportional to the stock of health. If the individual is well, utility is given by the strictly concave and increasing function $U(C, \phi H)$, while utility in the event of illness is assumed to be $U(0, \phi H)$. (U[Death] $\equiv 0$, as in the text.) If one also assumes that $U(C, \phi H)$ is separable, then the qualitative results obtained in Section IIB are unaltered by including H in the utility function.

subject to (3).[6] It is also reasonable to assume that $I \geq 0$ since health capital can neither be sold nor consumed.

In equation (6), T, the time of the individual's death, is assumed known with certainty and outside of his control. This rather restrictive assumption is relaxed below; however, the case in which T is fixed provides an interesting basis for comparing investment in health with investment in human capital.

B. *Life-Cycle Behavior of Investment and Health Capital*

In what follows I shall describe the paths of investment and health capital by examining the necessary conditions corresponding to equation (6). Before doing so, however, it should be established that the path of investment which maximizes (6) is consistent with the amount of investment which the individual would choose at each t based on the most recent information available to him. I shall argue that the path of planned investment is consistent with the actual path of investment if the distribution $F(\bar{H})$ is regarded as an objective distribution and if purchases of consumption goods at instant t must be made ε minutes before the individual knows whether or not he is ill.[7]

Suppose, for example, that the individual revises his investment decision at time τ, given knowledge of the path $\bar{H}_t$, $0 \leq t < \tau$. If $F(\bar{H})$ is regarded as the true distribution from which a value of $\bar{H}$ is drawn at each t, then the individual has no reason to alter $F(\bar{H})$ at time τ. If it is also assumed that the individual must order the goods which he will consume at τ an arbitrarily small amount of time before $\bar{H}_\tau$ is revealed, then the individual at τ will solve the problem

$$\max_{I_t} \int_\tau^T e^{-\rho(\tau-t)} F(H) U(Y - pI)\, dt, \tag{7}$$

given H_τ, and the values of C_τ and I_τ chosen at τ will not differ from the values of C_τ and I_τ chosen at time 0.

The paths of investment and health capital may now be examined by writing the Hamiltonian function and necessary conditions corresponding

[6] If the instantaneous utility function $U(C)$ is bounded from below, then by Tonelli's theorem it can be shown that equation (6) is equivalent to the objective function suggested by the expected utility hypothesis,

$$E\left[\int_0^T e^{-\rho t} V(t)\, dt\right],$$

where

$$V(t) = \begin{cases} U(C_t) & \text{if } H_t \geq \bar{H}_t \\ 0 & \text{if } H_t < \bar{H}_t \end{cases}.$$

[7] There are many cases in which this assumption seems to fit reality. Food and clothing, for example, are goods which are usually purchased prior to the instant of consumption and cannot easily be resold.

to equation (6). Omitting the discount factor, the current-value Hamiltonian becomes

$$\mathscr{L} = F(H)U(Y - pI) + \mu(I - \delta H) + rI, \tag{8}$$

where μ is the multiplier attached to (3) and r corresponds to the constraint $I \geq 0$. For any H and μ, I must maximize $\mathscr{L}$, hence (9) must hold along an optimal path. Equations (10)–(13) must also be satisfied.

$$\partial\mathscr{L}/\partial I = -F(H)pU'(Y - pI) + \mu + r = 0, \tag{9}$$

$$\dot{\mu} = (\rho + \delta)\mu - F'(H)U(Y - pI), \tag{10}$$

$$\dot{H} = I - \delta H, \tag{11}$$

$$e^{-\rho T}\mu_T H_T = 0, \tag{12}$$

$$r \geq 0, rI = 0. \tag{13}$$

Since equations (9)–(13) play an important role in subsequent analysis, it is well to examine them before proceeding. In these equations, μ, the shadow price of health, represents the increase in future expected utility of a unit increase in the stock of health capital. Equation (9), which determines I as a function of μ and H, states that the individual will increase current consumption at the expense of investment in health capital until the expected marginal utility of a unit of current consumption just equals μ, the future utility foregone by not investing in health.

Equation (10), which may also be written

$$F'(H)U(Y - pI) = \left(\rho + \delta - \frac{\dot{\mu}}{\mu}\right)\mu, \tag{14}$$

is a condition which must be satisfied by the optimal stock of health capital. In interpreting (14) recall that for the individual, investment in health is analogous to investment in a marketable capital good such as a consumer durable. What the equation says is that $F'(H)U(Y - pI)$, the increase in utility obtained by purchasing an additional unit of H for an instant, must equal $[\rho + \delta - (\dot{\mu}/\mu)]\mu$, the cost of an instantaneous increase in H. This cost is comprised of three parts—the psychic return ρ foregone by "spending" μ on H rather than on current consumption; the depreciation cost $\delta\mu$, reflecting the instantaneous deterioration in H; and $\dot{\mu}$, the "capital gain" which accrues to the holder of H. (The latter must be subtracted from $[\rho + \delta]\mu$ to arrive at the cost of capital.) Equation (10) is thus analogous to the familiar result from capital theory that the marginal product of capital must equal the supply price of capital—interest plus depreciation minus capital gains. The difference here is that because the stock of health is not marketable, all returns must be measured in utility terms.

Equations (9)–(11) must be satisfied by I, H, and μ at all points along an optimal path. Equation (12), on the other hand, places restrictions on these variables at the terminal date T. Equation (12) indicates that at the time of his death an individual will optimally have used up his stock of health capital or will be in a position where further increases in the stock of health cannot increase utility ($\mu_T = 0$). It is clear, however, from (11) and the fact that I must be nonnegative, that the individual will never be able to deplete his stock of health capital in finite time. It therefore follows that the shadow price of health must be zero at the time of death. Since μ is continuous, this implies that $\mu < F(H)pU'(Y)$ for some interval prior to T, and hence that $I = 0$ prior to the time of death.

Thus as long as the time of death is known with certainty, it must be preceded by a period in which no expenditures on preventative health care are made. This observation also implies that the individual's health stock will be decreasing steadily toward the end of his life and that illness is likely to occur more frequently prior to death.

We shall now examine how investment in health and health capital change over the individual's lifetime. Equations (9)–(13) do not exclude the possibility that the individual never invests in health; however, the case of $I_t > 0$ (except at the time of death) is clearly more interesting. When $I_t > 0$ the rate of change in investment, $\dot{I}$, may be derived by differentiating equation (9) with respect to time and then substituting for $\dot{\mu}$ and $\dot{H}$ from equations (10) and (11).

$$\dot{I} = \frac{-U'}{pU''}\left[\delta + \rho - \frac{F'(H)}{F(H)}\left(\frac{U}{pU'} + I - \delta H\right)\right]. \tag{15}$$

Equations (11) and (15), when solved simultaneously, yield a family of I-H paths, one for each value of T. The properties of these paths are examined in figure 1 by plotting the stationary loci $\dot{I} = 0$ and $\dot{H} = 0$.

If $\dot{H}$ is set equal to zero, the result is a straight line with slope $1/\delta$. For values of I to the left of the line, $\dot{H}$ is decreasing while $\dot{H} > 0$ to the right. The shape of $\dot{I} = 0$ is, however, more complicated. For $I > 0$ the locus $\dot{I} = 0$ may conveniently be written

$$\begin{aligned} \frac{U(Y - pI)}{pU'(Y - pI)} + I &= \frac{\delta + \rho}{\xi(H)} + \delta H \\ &\equiv J(H), \end{aligned} \tag{16}$$

where $\xi(H) \equiv F'(H)/F(H)$ is the conditional probability density of illness. Loosely interpreted, this is the probability that the individual falls ill when his health stock is H_t, given that he is well when his health capital exceeds H_t. We shall assume, as seems reasonable, that the conditional probability of illness is a decreasing function of H and that

$$\infty \geq \lim_{H \to 0} \xi(H) > 0, \quad \text{and} \quad \lim_{H \to \infty} \xi(H) = 0. \tag{17}$$

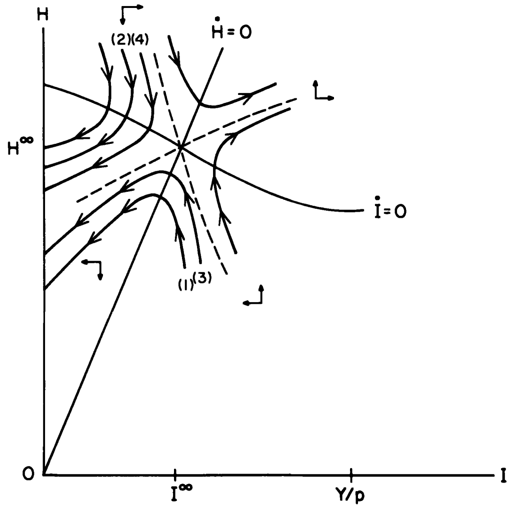

FIG. 1.—Paths of investment and health capital when death is exogenous

In view of this assumption, the right-hand side of (16) is a strictly increasing function of H. As the reader may verify, the left-hand side of the equation is a decreasing function of I for $0 \leq I \leq Y/p$ and an increasing function of I for $I > Y/p$.[8] The locus $\dot{I} = 0$ therefore begins at the point $\{0, J^{-1}[U(Y)/pU'(Y)]\}$, slopes downward to the point $[Y/p, J^{-1}(Y/p)]$, and increases thereafter. It can also be shown that the loci $\dot{H} = 0$ and $\dot{I} = 0$ must intersect only once, and that this intersection must occur on the downward-sloping portion of $\dot{I} = 0$.[9]

We are now in a position to describe the optimal paths of investment and health capital. In figure 1 the heavy solid lines represent solutions to the system (11)–(15) for various values of T. Paths such as (3) and (4), which more closely approach the stationary values I^∞ and H^∞ than do paths (1) and (2), correspond to larger values of T. Since investment in health must be zero at the time of death, any path which does not terminate on the H-axis cannot be optimal. This implies that an optimal path will resemble (1) or (3) if H is initially small ($H_0 < H^\infty$) and (2) or (4) if H_0 is large ($H_0 > H^\infty$).

[8] Specifically, the derivative with respect to I of the left-hand side of (16) is $UU''/p(U')^2$, which is negative for $U > 0$ and positive for $U < 0$.

[9] Whenever the curves $\dot{I} = 0$ and $\dot{H} = 0$ intersect, it must be the case that $I = \delta H$ and $\delta + \rho = \xi(H)[(U/pU') + I - \delta H]$. Substituting the first equation into the second and rearranging terms, $\xi(H) = (\delta + \rho)p[U'(Y - p\delta H)]/[U(Y - p\delta H)]$, which may be solved for H. The left-hand side of the equation is, by assumption, monotone decreasing in H, while the right-hand side is strictly increasing in H. It thus follows that the stationary loci can intersect only once. Since the right-hand side of the equation approaches infinity as H approaches $Y(\delta p)^{-1}$, this implies that $H^\infty < Y(\delta p)^{-1}$ and hence that $I^\infty < Y/p$. Hence the phase diagram of the system must appear as is figure 1.

"Unhealthy" individuals (those for whom $H_0 < H^\infty$) will therefore build up their stock of health capital during their early and middle years and let it decay during old age. This will be achieved by investing heavily in health in the early years, presumably via nutrition and exercise, with I decreasing steadily throughout the individual's lifetime and reaching zero sometime before death.

What is perhaps a more typical case occurs if the individual is in good health at the beginning of his lifetime ($H_0 > H^\infty$). The optimal plan in this instance is to allow the stock of health capital to decrease steadily over time. The corresponding path of investment will not, however, be monotone decreasing. For T sufficiently large, the optimal strategy is to increase the amount invested in health to a peak in the early years of life and let it decline steadily thereafter. This initial buildup of I is necessary to insure that the health stock does not decline too fast for long-lived individuals.

The results of figure 1 suggest an interesting parallel between investment in health capital and investment in human capital. In the human-capital literature (see Blinder and Weiss 1976), the time devoted to acquiring productive skills usually declines over the life cycle as the period over which returns are received decreases. This phenomenon also occurs above. Investment in health decreases as death approaches and the marginal benefits from increasing the stock of health capital (μ) decline. Regarding the behavior of human capital, the individual usually builds up his stock of productive skills early in life, with the stock reaching a peak in the middle years and decreasing thereafter. This is also true of health capital if the initial endowment of health is low ($H_0 < H^\infty$). However, since the decision to invest in health is presumably made during adolescence or early adulthood, it is very likely that the stock of health capital at that time will be high and will decline steadily throughout the individual's lifetime.

It should also be noted that the paths of investment in figure 1 contrast sharply with the results obtained by Grossman (1972) regarding life-cycle behavior of investment in health. A point emphasized by Grossman is that investment in health will be positively correlated with age as long as the elasticity of the marginal efficiency of health capital is less than one. This conclusion is reasonable if investment in health is interpreted to mean expenditure in the event of illness, since individuals are more likely to be ill when they are old than when they are young. It is less likely, however, that *preventative* medical expenditures will increase over time, since investments made late in life yield returns over a shorter period than investments made early in life. In the context of Grossman's model a portion of investment in health may be interpreted as recuperative expenditure, and hence it is reasonable that investment in his model should increase over time. In my model, on the other hand, investment in health must be interpreted solely as preventative health care, since expenditure in the event of illness is a random event. Hence it is reasonable that I decreases over the life cycle.

C. Investment in Health When Death Is Endogenous

It should be remembered that all of the preceding results rest on the somewhat restrictive assumption that the stock of health capital in no way influences the time of death. A more reasonable assumption is that death occurs when the health stock falls below some level $\hat{H}$, where $\hat{H} \leq \bar{H}$. This definition of death requires, however, that the assumptions regarding $\bar{H}$ be altered slightly. It is now assumed that $\bar{H}$ is independently and identically distributed for all t over the interval $[\hat{H}, \infty)$.[10] Death, which occurs when H_t falls below $\hat{H}$, is therefore equivalent to being sick 100 percent of the time. This assumption may conflict with the observation that some persons prefer being invalids to dying; however, it is hard to define death in a more "realistic" way which is also analytically convenient.

If this definition of death were incorporated into the model above, T would be determined by the condition $H_T = \hat{H}$, and the individual would clearly choose to live forever. The choice of a finite lifetime can, however, be explained by allowing the rate of depreciation of the human capital stock to increase as the individual ages, a device first employed by Grossman (1972). If the health stock decays fast enough, desires for present consumption, together with the individual's budget constraint, will cause him to live a finite life.

Unfortunately, if δ is a function of time, the model of Sections A and B is no longer autonomous and the analysis becomes complicated. It is nevertheless possible to say something about the paths of investment and health capital for the case in which the individual is risk neutral. Let us therefore assume that the individual solves the control problem

$$\max_{I_t, T} \int_0^T e^{-\rho t} F(H)(Y - pI)\, dt \tag{18}$$

subject to the constraints $\dot{H} = I - \delta H$, $0 \leq I \leq Y/p$, and $H_T = \hat{H}$.

Since the Hamiltonian function

$$\mathscr{L} = F(H)(Y - pI) + \psi(I - \delta H) \tag{19}$$

[10] The expression $F(H)$ is now redefined as

$$\int_{\hat{H}}^{H_t} f(\bar{H})\, d\bar{H}.$$

is now linear in I, the individual will set

$$\begin{aligned} I &= 0 \quad \text{if} \quad \psi - pF(H) < 0, \\ I &= \frac{Y}{P} \quad \text{if} \quad \psi - pF(H) > 0, \end{aligned} \tag{20}$$

where ψ is the value of a unit increase in H and $\psi - pF(H)$ may be interpreted as the net yield, measured in utility terms, of an additional unit of investment. In the intermediate case in which $\psi - pF(H)$ is zero, any value of I in the interval $[0, Y/p]$ is possible. The optimal I is determined in this case by differentiating $\psi - pF(H)$ with respect to time,

$$\frac{d}{dt}[\psi - pF(H)] = \dot{\psi} - pF'(H)\dot{H} = 0, \tag{21}$$

and substituting for $\dot{H}$ and $\dot{\psi}$ from equations (3) and (22), both of which must hold along an optimal path. The resulting equation, (23), may be solved for

$$\dot{\psi} = \psi(\delta + \rho) - F'(H)(Y - pI), \tag{22}$$

$$\frac{p(\delta + \rho)}{Y - p\delta H} = \xi(H), \tag{23}$$

and the optimal health stock, H_t^*, and investment, I_t^*, may be found in turn from the state equation $I = \delta H + \dot{H}$.[11]

The important question, of course, is how H_t^* and I_t^* vary over the life cycle. As in Section IIB the analysis below is confined to the case in which $I_t > 0$, except at the time of death. To describe the behavior of health capital over time, equation (23) is graphed in figure 2. The left-hand side of the equation is monotone increasing in H and discontinuous at $H = Y(p\delta)^{-1}$, while the right-hand side is a decreasing function of H which either intersects or is asymptotic to the line through $\hat{H}$. Since initially δ will be small, it is likely, unless the cost of investment is very high, that $Y(p\delta)^{-1}$ will exceed $\hat{H}$, thus guaranteeing a unique solution for H^*. As the individual ages and his health capital decays at a faster rate, $Y(\delta p)^{-1}$ grows smaller and $p(\delta + \rho)(Y - p\delta H)^{-1}$ shifts graphically upward. Thus, as expected, the optimal stock of health capital decreases monotonically, with the precise rate of decrease given by

[11] In determining the paths I_t^* and H_t^*, a problem arises if the solution to (23) is not unique or if the value of I_t^* implied by (23) violates the constraint $0 \leq I \leq Y/p$. Fortunately, the assumption $\xi'(H) < 0$ guarantees that H_t^* is unique. It can also be shown that $I^* < Y/p$ (see n. 12 below); however, the possibility that $I_t^* = 0$ cannot be ruled out.

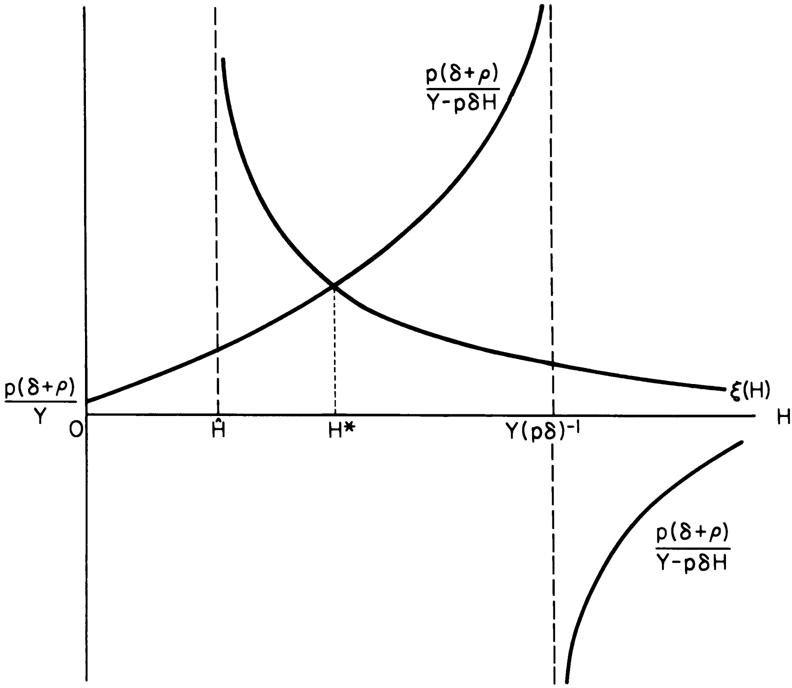

FIG. 2.—Determination of the optimal stock of health capital when death is endogenous

equation (24):[12]

$$\dot{H}^* = \frac{\dot{\delta} p[H\xi(H) + 1]}{\xi'(H)(Y - p\delta H) - \xi(H)\delta p}. \tag{24}$$

As long as δ continues to increase over time, $p(\delta + p)(Y - p\delta H)^{-1}$ must eventually intersect $\xi(H)$ at $H = \hat{H}$, or, if $\xi(H)$ is asymptotic to $\hat{H}$, $\hat{H}$ must exceed $Y(\delta p)^{-1}$, implying that an optimal H^* does not exist. In either case, death occurs in finite time. It should also be noted that the proportion of time that the individual will spend ill increases as the individual ages, due to the fact that as the stock of health capital falls, the probability of being ill at any instant rises.

As indicated above, once the optimal stock of health capital is known, investment in health may be obtained by solving equation (3) for I^*,

$$I^* = \dot{H}^* + \delta H^*.^{13} \tag{25}$$

[12] An important result of figure 2 is that $I_t^* < Y/p$, which guarantees that the individual always has a positive level of consumption. As figure 2 implies, $\delta H_t^* < Y/p$; however, if $\dot{H}_t^* < 0$, $I_t^* < \delta H_t^*$, implying $I_t^* < Y/p$.

[13] Equation (25) holds, strictly speaking, only when $I > 0$. If $I = 0$, equations (23) and (24) no longer apply and the individual is said to be in a blocked interval. However, conditions may be placed on the path of δ through time to guarantee that the individual will never be in blocked interval, except at the time of death.

The behavior of I^* over the life cycle is then determined by differentiating (25) with respect to time,

$$\dot{I}^* = \ddot{H}^* + \dot{\delta}H^* + \delta\dot{H}^*. \tag{26}$$

Unfortunately, previous assumptions no longer guarantee that $\dot{I}$ is unambiguously negative after some point in the life cycle, as was the case in IIB. The explanation for this is simple: Even though marginal benefits from increasing the stock of health capital, ψ, decline over time, replacement investment, δH_t, may increase as a result of $\dot{\delta} > 0$. Hence it is no longer necessarily the case that $\dot{I} < 0$.

Before leaving figure 2 it is interesting to examine how the stock of health capital varies among individuals in different income classes and among individuals living in different environments. An immediate implication of the diagram is that among individuals with different incomes who are otherwise identical, wealthier individuals will maintain higher stocks of health capital than poorer persons. This implies, through the function $1 - F(H)$, that low-income individuals, ceteris paribus, will spend more time ill than their wealthier counterparts.

A less obvious implication of equation (23) is that persons who live in polluted environments will choose to maintain lower stocks of health capital and, for this reason, will be ill more often than individuals who live in healthier environments. One way to capture the effects of pollution is to say that health capital deteriorates faster, that is, that δ_t is higher, the more polluted the environment. This implies that pollution has a cumulative deleterious effect on health which, if not offset by investment in health, will cause individuals to be ill more often and die sooner in a high-pollution area. Since $\partial H_t^*/\partial \delta_t < 0$, the model shows that individuals will not fully offset the effects of pollution by investing in health, and hence that the optimal health stock will be lower the more polluted the environment.

Finally, the model suggests that individuals who are more likely to become ill will compensate for this fact by maintaining higher stocks of health capital. In the language of IIA, individuals who have a higher probability of being ill (for a given H) have distribution functions $F(\bar{H})$ which lie below those of individuals who are less likely to be ill. To see how this affects the optimal stock of health capital, let us consider the family of density functions

$$f(\bar{H}) = \begin{cases} \alpha e^{-\alpha \bar{H}}, & \bar{H} \geq 0 \\ 0, & \bar{H} < 0 \end{cases}, \quad \alpha \geq 1.^{14} \tag{27}$$

This family satisfies the condition $\xi'(H) < 0$, and has the intuitively appealing property that small values of $\bar{H}$ are more likely to occur than

[14] Without loss of generality, H is assumed to be measured as a deviation from $\hat{H}$.

large values of $\bar{H}$. The corresponding distribution function is given by

$$F(H) = 1 - e^{-\alpha H}. \tag{28}$$

Since

$$\frac{\partial F(H)}{\partial \alpha} = He^{-\alpha H} \tag{29}$$

is positive for $H > 0$, an individual for whom α is small has a lower probability of being well for any H than a person for whom α is large.

The optimal health stock depends, however, on $\xi(H)$, the conditional probability density of illness. For the distribution function (28),

$$\xi(H) = \frac{\alpha e^{-\alpha H}}{1 - e^{-\alpha H}} \tag{30}$$

and

$$\frac{\partial \xi(H)}{\partial \alpha} = \frac{(1 - \alpha H)(1 - e^{-\alpha H})e^{-\alpha H} - \alpha He^{-\alpha H}}{(1 - e^{-\alpha H})^2}, \tag{31}$$

which is negative for $H \geq \alpha^{-1}$. Thus in terms of figure 2, $\xi(H)$ will be higher (except for small values of $H - \hat{H}$) the lower is α. This implies, in particular, that individuals who are more likely to be ill will maintain higher stocks of health capital.

III. Occupational Choice as Investment in Health

After analyzing what is conventionally meant by investment in health, let us now examine a more subtle way in which an individual affects his stock of health capital—namely, through choice of occupation. In this section the problem facing the individual is one of trading higher wages for greater exposure to a pollutant which has deleterious effects on the worker's health and increases the probability that he will die at an early age. This type of trade-off is especially important in industries such as coal mining and asbestos manufacturing; indeed, a major justification for the present model is the insights which it yields into the decision to work in such occupations.

A. The Model

In order to focus on the problem of occupational choice, we shall assume at each t that the individual has $\bar{L}$ of labor which he divides between a "healthy" occupation (farming) and an "unhealthy" occupation (coal mining), which pays a higher wage but exposes him to a potentially lethal pollutant. If L_t denotes the number of hours worked in the unhealthy occupation, then the individual's income at t may be written

$$Y_t = \bar{L} + wL_t, \tag{32}$$

where w represents the wage differential between the two occupations and the wage in the healthy occupation is taken without loss of generality to be one. As the individual works in the unhealthy occupation he is assumed to absorb a pollutant, such as coal dust, at a rate of b units per hour. The rate of change in the pollution stock is thus given by

$$\dot{P} = bL - \alpha P, \tag{33}$$

where α is the rate at which the stock decays.

Income and the pollution stock affect the individual's utility in the following way. If the individual is alive at time t, he receives utility from income according to the strictly concave and increasing function $U(Y)$, where $U(\cdot)$ has the properties listed in (4). If the individual is dead, income is zero and no utility is received. Although by assumption the pollutant has no immediate effect on utility, a sufficiently high concentration of the pollutant will cause death.[15] Formally, death occurs if $P \geq \bar{P}$, where $\bar{P}$ is the lethal dose of the pollutant. We shall assume that there exists a true $\bar{P}$ which does not change throughout the individual's lifetime but which is unknown to the individual. Subjective beliefs about $\bar{P}$ at time 0 are reflected in the probability density function $g(\bar{P})$, which is distributed over the interval (P_0, ∞) with $g(\bar{P}) > 0$.[16] As time passes, the knowledge that the individual has not yet died provides information which may be used to update the distribution $g(\bar{P})$. It is shown in Appendix B that if the worker revises his beliefs about $\bar{P}$ at some time τ, $\tau > 0$, the path of employment based on these updated beliefs will be consistent with the path of employment chosen at 0.

The above assumptions imply that the probability that the individual dies at t is $G(P_t)$, where $G(\cdot)$ is the cumulative distribution corresponding to $g(\bar{P})$. However, for the purpose of computing expected utility, it is not the probability that the individual dies at t but the probability that he is alive at t which is of interest. The probability that the individual is alive at t is computed in Appendix A by noting that the event "the individual is alive at t" is equivalent to the event "the individual does not die prior to t." As is shown below, the probability that the individual does not die prior to t is the probability that $\bar{P}$ exceeds the *largest* value of P in the interval $[0, t]$. Hence

$$P(\text{alive at } t) = \int_{\max\limits_{0 \leq \tau \leq t} P_\tau}^{\infty} g(\bar{P})\, d\bar{P} \equiv \Lambda\left(\max_{0 \leq \tau \leq t} P_\tau\right). \tag{34}$$

By controlling the size of his pollution stock the individual thus affects the probability that he will live or die and in turn determines his expected

[15] Many industrial pollutants do, of course, have immediate side effects, which suggests that utility should depend on P as well as on Y. Unfortunately, the case where $U = U(Y, P)$ is intractable, for reasons that will become clear below.

[16] Since it is assumed that $\bar{P}$ remains constant throughout the individual's lifetime, the knowledge that the individual is alive at $t = 0$ implies that $P_0 < \bar{P}$.

utility,

$$EU_t = U(Y_t)\Lambda\left(\max_{0 \leq \tau \leq t} P_\tau\right). \tag{35}$$

The interesting question is how the relationship between the pollution stock and expected utility affects the individual's choice of occupation.

Formally, the problem of occupational choice consists of selecting a path L_t to maximize discounted expected utility

$$\int_0^T e^{-\rho t}U(Y)\Lambda\left(\max_{0 \leq \tau \leq t} P_\tau\right) dt \tag{36}$$

subject to (33) and to the inequality constraint $\bar{L} \geq L \geq 0$. As equation (36) implies, the individual is assumed to die at T from "natural causes" if he does not die sooner as a result of his job.

The analysis of equation (36) is complicated by the fact that instantaneous expected utility depends not only on the current value of the pollution stock but on all previous values of P as well. However, the objective function may be simplified by showing that the optimal pollution stock never decreases, and hence that

$$\Lambda\left(\max_{0 \leq \tau \leq t} P_\tau\right) = \Lambda(P_t).$$

To see this, note that in equation (36) P affects utility only by altering the probability that the worker is alive. Since the probability that the worker is alive at t is the probability that $\bar{P}$ exceeds the largest value of P in the interval $[0, t]$, the individual can never increase his utility at t, or at any time following t, by setting $\dot{P}_t < 0$. Since this argument holds for any t, the optimal pollution stock can never decrease and

$$\Lambda\left(\max_{0 \leq \tau \leq t} P_\tau\right) = \Lambda(P_t). \tag{37}$$

With this simplification, equation (36) may be written

$$\int_0^T e^{-\rho t}\Lambda(P)U(\bar{L} + wL)\, dt, \tag{38}$$

and the problem of occupational choice is seen to be analogous to the problem of investment in health studied in Section IIB.

B. Life-Cycle Behavior of Employment and Pollution

Note that in simplifying (36) an important result has been obtained, namely, that P_t never decreases. While intuition might suggest that an individual working in a risky occupation would build up his pollution stock when he was young, allow it to decrease later on in life, and possibly return to the risky occupation in old age, the model indicates that as long as the individual suffers no immediate discomfort from the pollutant, nothing is gained from allowing P to decrease since decreasing P cannot increase the probability that the individual will live. This suggests that individuals will take "recuperative breaks" from an unhealthy occupation only if the pollutant decreases their instantaneous utility. The fact that the job increases the probability of death is not, by itself, sufficient reason for leaving the occupation.

A related question of interest is how L_t, the path of employment in the risky occupation, varies over the life cycle. Two cases may be distinguished. If initially $P_0 = 0$ and $L_0 = 0$, then it may be shown that the individual will never work in the risky occupation.[17] On the other hand, if the wage is high enough that $L_0 > 0$, equation (33) and the condition $\dot{P} \geq 0$ imply that the individual will always work some amount of time in the risky occupation. The path of employment in the latter case may be derived from the results of Section IIB by noting that a high stock of pollution is equivalent to a low health stock, while an increase in investment in health ($\dot{I} > 0$) corresponds to a decrease in the number of hours worked in the risky occupation ($\dot{L} < 0$).[18] Since $\dot{P} \geq 0$ ($\dot{H} \leq 0$), only paths analogous to (2) and (4) in figure 1 are possible.[19] Thus employment in the risky occupation will begin at $L_0 > 0$, decrease as the individual grows older, and then increase as retirement (time T) approaches. This implies that an individual who chooses to work in an unhealthy occupation will expose himself to large amounts of pollution early in life, decreasing his exposure in middle age to prevent too rapid an accumulation of the pollutant. As the individual grows older, the negative effects of pollution last for a shorter length of time and employment in the risky occupation increases. Thus, even though the optimal pollution stock never decreases, the amount of time worked in the risky occupation does decrease during the life cycle to prevent P from increasing too rapidly.

[17] The condition which determines L as a function of λ and P is $wU'(Y)\Lambda(P) + b\lambda \leq 0$. Hence if $[wU'(L)/b] < -\lambda_0$, $L_0 = 0$. As long as $L_t = 0$, however, it can be shown that λ decreases over time and hence that $-\lambda_t$ increases in absolute value. Hence if $L_0 = 0$ and $P_0 = 0$, $L_t = 0$ thereafter.

[18] For a complete analysis of the solution to (38) the reader is referred to Cropper (1976).

[19] The fact that paths corresponding to (1) and (3) are no longer possible implies that there is an important difference between the models of Sections II and III. The difference is in the nature of the randomness in each model. In Section II the probability that the individual is well at t is, by assumption, independent of the probability that he is well at t'. Hence by setting $\dot{H} < 0$, the individual can always increase the probability of being well. In Section III, however, the probability of being alive at t is not independent of the probability of being alive at t'; in fact, P(alive at t) $\leq$ P(alive at t'), where $t' < t$. Thus the individual can never increase the probability that he is alive at t by setting $\dot{P}_t < 0$. For this reason $\dot{P} \geq 0$, and one of the two strategies which is possible in Section II is ruled out in Section III.

In addition to the above conclusions, the present model has interesting implications for an important real-world problem. Frequently, it is learned that a job previously thought to be harmless actually exposes workers to a dangerous pollutant. An important question is how workers should optimally react to this information, assuming they have already ingested significant amounts of the pollutant but also assuming that they suffer no noticeable side effects from it. A commonly held view is that workers should leave the unhealthy occupation in an attempt to neutralize the effects of the pollutant. In many cases, however, workers are observed to remain in risky occupations after being warned of the dangers involved. The model suggests that this behavior is indeed optimal. By analogy with figure 1, when T is "small" the worker will at first reduce the number of hours worked in the unhealthy occupation; however, L will eventually increase, with the individual working full time in the risky occupation as retirement approaches. Thus the individual's failure to decrease his exposure to risk need not be the result of myopia or lack of employment opportunities but may reflect a rational decision.

IV. Conclusion

This paper has presented two models of investment in health which explicitly recognize the random nature of illness and death. In the first model the demand for preventative medical care (checkups, dietary supplements, etc.) is derived from the more basic demand for health capital. Individuals desire to increase their stock of health capital in order to decrease the probability of illness. This is because an individual, when ill, receives no utility from consumption; hence investment in health increases expected utility of consumption. Against this increase in expected utility the individual must balance the cost of investing in health, which depends on the price of medical services, the rate of depreciation of health capital, and the individual's subjective discount rate.

The important question, of course, is how investment in health and the optimal health stock vary throughout the life cycle. In human-capital theory, individuals usually invest most heavily in themselves when they are young, with the amount of time devoted to training declining steadily thereafter (see Blinder and Weiss 1976). If the time of death were known with certainty, one would expect that investment in health would also be concentrated at the beginning of one's life and decrease as the time of death approached. If the conditional probability of illness is a decreasing function of the health stock, this is in fact the case. However, a major difference between investment in human capital and investment in health is that in a model of health capital the time of death cannot be treated as exogenous. This problem is remedied in Section II by assuming that death occurs if the individual is ill 100 percent of the time. If one also assumes that the rate of decay of health capital increases with age, then it can be shown that the individual will live a finite life and that his stock of health capital will decrease as the time of death approaches. This, in turn, implies that the fraction of time spent ill increases as the individual ages and that recuperative medical expenditures should, therefore, increase. Preventative medical expenditures, on the other hand, may decrease as the individual ages.

In the second model, investment in health is related to occupational choice. Casual observation suggests that for individuals with a fixed stock of human capital there is often a trade-off between earning higher wages and working in an unhealthy environment. This trade-off is especially important when the higher-paying job exposes the worker to an industrial pollutant such as asbestos, which in sufficiently high concentrations can cause death. In the model of Section III the individual must choose between a "safe" occupation which has no effects on health and a higher-paying occupation which exposes the worker to a potentially lethal pollutant. Increases in the stock of the pollutant increase the probability of death; hence the worker must trade the opportunity to earn a higher income against an increased probability of dying.

If it is assumed that the pollutant does not yield immediate disutility to the worker, it can be shown that the optimal pollution stock never decreases and that the worker if he ever works in the risky occupation will always expose himself to some amount of pollution. This implies, in particular, that an individual who has been working in an occupation which is discovered to be hazardous to the health need not quit the occupation once he is informed of the dangers to his health. Thus what upon casual observation may appear to be irrational behavior is shown by our model to be consistent with rational decision making.

Appendix A

To compute the probability that the individual is alive at time t, let us define the event, "the individual does not die at τ" and denote it $\bar{D}_\tau$:

$$\bar{D}_\tau = \{\bar{P} \mid \bar{P} \geq P_\tau\}. \tag{A1}$$

Since the probability that the individual is alive at t is the probability that he has not died at any time up to and including t,

$$P(\text{alive at } t) = P\left\{\bigcap_{0 \leq \tau \leq t} \bar{D}_\tau\right\}. \tag{A2}$$

Note, however, that because the lethal dose of the pollutant does not change throughout the individual's lifetime,

$$\bigcap_{0 \leq \tau \leq t} \bar{D}_\tau = \left\{\bar{P} \mid \bar{P} \geq \max_{0 \leq \tau \leq t} P_\tau\right\}. \tag{A3}$$

Thus

$$P\left(\bigcap_{0\le\tau\le t} \bar{D}_\tau\right) = \int_{\max\limits_{0\le\tau\le t} P_\tau}^{\infty} g(\bar{P})\, d\bar{P}. \tag{A4}$$

Hence if

$$\Lambda(P) \equiv \int_{P}^{\infty} g(\bar{P})\, d\bar{P},$$

$$P\left(\bigcap_{0\le\tau\le t} D_\tau\right) = \Lambda\left(\max_{0\le\tau\le t} P_\tau\right). \tag{A5}$$

Appendix B

The purpose of this Appendix is to show that the optimal path of employment in Section III allows for revision of expectations about $\bar{P}$, the lethal dose of the industrial pollutant. The method of proof follows Long (1975).

Let $g(\bar{P} \mid P_\tau)$ be the density function over $\bar{P}$, given that P_τ of the pollutant has already accumulated and that death has not yet occurred.

One may likewise define

$$\Lambda(P_t \mid P_\tau) \equiv \int_{P_t}^{\infty} g(\bar{P} \mid P_\tau)\, d\bar{P} \tag{B1}$$

to be the probability that the lethal dose is at least of size P_t, given that P_τ of the pollutant has accumulated to date.

The question is whether the path L_t, which maximizes

$$\int_0^T e^{-\rho t}\Lambda(P_t \mid 0) U(\bar{L} + wL)\, dt \tag{B2}$$

subject to

$$\begin{aligned} &\dot{P} = bL - \alpha P, \\ &\bar{L} \ge L \ge 0, \\ &P_0 \text{ given,} \end{aligned} \tag{B3}$$

is consistent with the path chosen at time t', provided death has not yet occurred, to maximize

$$\int_{t'}^T e^{-\rho(t'-t)}\Lambda(P_t \mid P_{t'}) U(\bar{L} + wL)\, dt \tag{B4}$$

subject to (B3).

Since $\Lambda(P_t, P_{t'} \mid 0)$, the joint probability that $\bar{P} > P_t$ and $\bar{P} > P_{t'}$, equals $\Lambda(P_t \mid 0)$ for $P_t > P_{t'}$, expectations will be revised according to

$$\Lambda(P_t \mid P_{t'}) = \frac{\Lambda(P_t \mid 0)}{\Lambda(P_{t'} \mid 0)}. \tag{B5}$$

It thus follows that, given $P_{t'}$, the revised probability of being alive differs from the initial probability only by a multiplicative constant, and (B2) and (B4) will, therefore, yield identical solutions.

References

Blinder, Alan S., and Weiss, Yoram. "Human Capital and Labor Supply: A Synthesis." *J.P.E.* 84, no. 3 (June 1976): 449–72.

Cropper, M. L. "Two Models of Investment in Health." Working Paper no. 14, Univ. California, Riverside, 1976.

Grossman, Michael. "On the Concept of Health Capital and the Demand for Health." *J.P.E.* 80, no. 2 (March/April 1972): 223–55.

Long, Ngo Van. "Resource Extraction under the Uncertainty about Possible Nationalization." *J. Econ. Theory* 10 (February 1975): 42–53.

Thaler, Richard, and Rosen, Sherwin. "The Value of Saving a Life: Evidence from the Labor Market." In *Household Production and Consumption*, edited by Nestor E. Terleckyj. New York: Columbia Univ. Press (for Nat. Bur. Econ. Res.), 1975.